

10	4	6	3	5	9	2	1	7	8	11	0
4	10	0	9	11	3	8	7	1	2	5	6
2	8	10	7	9	1	6	5	11	0	3	4
5	11	1	10	0	4	9	8	2	3	6	7
3	9	11	8	10	2	7	6	0	1	4	5
11	5	7	4	6	10	3	2	8	9	0	1
6	0	2	11	1	5	10	9	3	4	7	8
7	1	3	0	2	6	11	10	4	5	8	9
1	7	9	6	8	0	5	4	10	11	2	3
0	6	8	5	7	11	4	3	9	10	1	2
9	3	5	2	4	8	1	0	6	7	10	11
8	2	4	1	3	7	0	11	5	6	9	10

Combinatorial set forms that preserve fixed dyads [4,10] and [1,7]

P10 I7

P4 I1

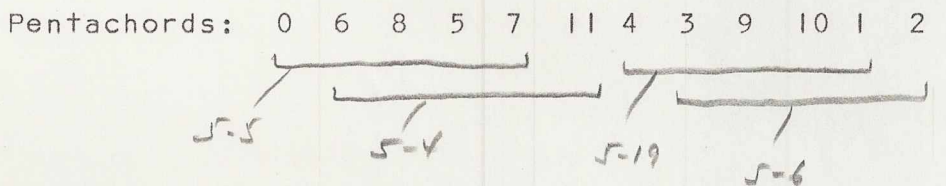
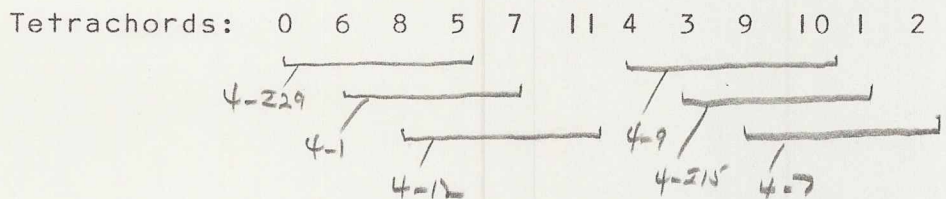
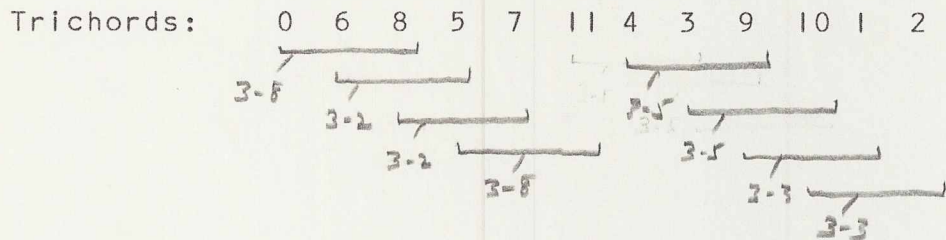
P7 I10

N.B. superscripts are pc integers that represent pitches as they appear in the score. I.e., P0 begins on pitch-class zero, row 3 from the bottom of the row matrix

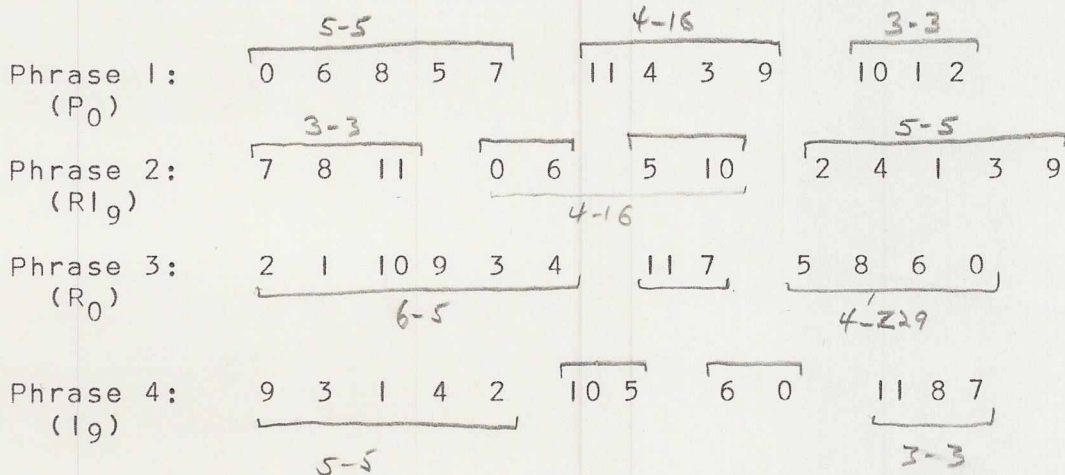
CYCLIC NOTATION - COMPARE I-comb. axes

Variations for Orchestra

Substructures of the row (contiguous subsets)



Actual partitioning of the theme does not observe hexachord boundaries:



5 + 4 + 3

→ 3 + 2 + 2 + 5

6 + 2 + 4

→ 5 + 2 + 2 + 3

Retrograde

Introduction

Set forms

m. 1-16.5: $\begin{cases} P_0 \\ I_9 \end{cases}$ combinatorial

m. 16.5-18.5 $\begin{cases} P_0 \\ I_0 I_0 \end{cases}$

m. 18.5-20 $\begin{cases} P_6 \\ I_6 \end{cases}$

m. 20-21.5 $\begin{cases} I_9 \\ RI_9 \end{cases}$ R_3
 P_3

m. 21.5-22 $\begin{cases} RI_3 \\ I_3 \\ RI_9 \\ I_9 \end{cases}$

M. 22-23.5 $\begin{cases} P_9 \\ R_9 \end{cases}$

m. 23.5-25 P_0
m. 25-26 **NB.** RI_1 $\left. \begin{array}{l} P_0 \\ RI_1 \end{array} \right\} \text{B-A-C-H here}$

m. 26-28 $\begin{cases} P_0 \\ I_9 \end{cases}$

m. 28-29 I_9

m. 29-30 $\begin{cases} R_0 \\ RI_9 \end{cases}$ combinatorial

m. 30 $\begin{cases} I_9 \\ P_0 \end{cases}$ combinatorial

m. 31-33 $\begin{cases} P_0 \\ I_9 \end{cases}$ combinatorial

Variations for Orchestra, Op.31

See Ruder, Werke and Maegaard

On p. 34 in Ruder is a very interesting annotation by Schoenberg relevant to the dates of composition (1926-28)

/
date of Webern Symphonies, Op. 21

Op.31 analyses

Leibowitz

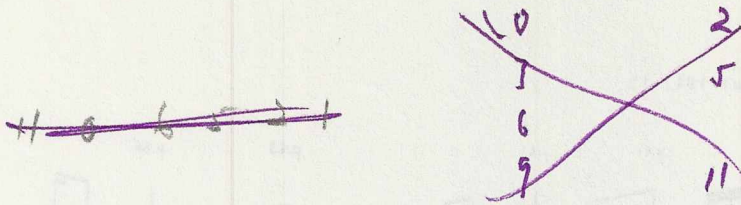
Rufer?

Dahlhaus (see 125a reading ~~xxxx~~ list) -- not in music library

Style & Idea, p. 131

P_3	3	9	11	8	10	2	7	6	0	1	4	5
P_0	0	6	8	5	7	11	4	3	9	10	1	2
I_9	9	3	1	4	2	10	5	6	0	11	8	7
I_6	6	0	10	1	11	7	2	3	9	8	5	4

) invariant digits (missed at combinatorial \mathbb{F} -level

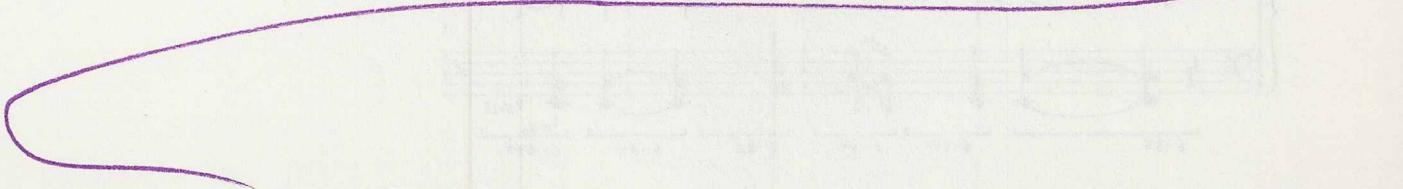


Q
6

Vav. IV

Accomp. {

P ₅	RI ₁₁	P ₇	I ₀	P ₁	RI ₁₀	P ₁₁	RI ₈	R ₉
I ₂	R ₂	I ₄	P ₃	I ₁₀	R ₁	I ₈	RI ₁₁	RI ₆



RI₂ R₂ RI₄ R₅ I₈ P₃ I₁₁ R₂ RI₇
 R₅ RI₁₁ R₇ RI₂ P₉ I₄ P₂ RI₁₁ R₁₀

P₂ I₇ R₁₀ RI₂ R₃ P₇ P₁ P₃ I₄ ||
 I₁₁ P₁₀ RI₇ R₅ RI₀ P₇ P₁ P₃ P₇ ||

58 59 60

<p>Pic Fl Ob 1. 2. EH Es Kl Kl 1.2. Bs Kl Fg 1.2.3 Kfg Hr 1. 2. Trp 1. 2. Hrf Cel Gg 1. 2. Br Vcl Kbs</p>	<p style="text-align: center;">3-4</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="padding: 2px;">1 - - - 1 8 0</td></tr> <tr><td style="padding: 2px;">10 - - - 10 5 9</td></tr> </table> <p style="text-align: center;">6-14</p> <p style="text-align: right; margin-right: 50px;"><i>order no</i></p> <p style="margin-right: 50px;">$(P-0^x 1,5,6)$</p> <p style="margin-right: 50px;">$(P-0^x 1,5,6)$</p> <p>$(P-0^x 1,2,3,4,5)$</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="padding: 2px;">10 4 - 4 6 3 - - - 5 -</td></tr> <tr><td style="padding: 2px;">10 4 - 4 6 3 - - - 5 -</td></tr> <tr><td style="padding: 2px;">10 4 - 4 6 3 - - - 5 -</td></tr> </table> <p style="text-align: right;">5-5</p> <p style="margin-right: 50px;">4 2 5</p> <p style="margin-right: 50px;">7 1 0</p> <p style="margin-right: 50px;">7 5 8</p> <p style="margin-right: 50px;">10 4 3</p> <p>$(I-0^x 2,3,4)$</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="padding: 2px;">4 2 5</td></tr> <tr><td style="padding: 2px;">7 5 8</td></tr> </table> <p style="text-align: right;">3-2</p> <p style="margin-right: 50px;">5-10</p> <p>$(I-3^x 2,3,4)$</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="padding: 2px;">7 7 9 6</td></tr> <tr><td style="padding: 2px;">4 4 6 3</td></tr> <tr><td style="padding: 2px;">7 7 9 6</td></tr> <tr><td style="padding: 2px;">4 4 6 3</td></tr> </table> <p style="text-align: right;">3-2 $(P-3^x 2,3,4)$</p> <p style="text-align: right;">5-10 $(P-0^x 2,3,4)$</p> <p style="text-align: center;">3-4</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="padding: 2px;">10 - 10 3 11</td></tr> <tr><td style="padding: 2px;">1 - 1 6 2 -</td></tr> </table> <p style="text-align: right;">6-14</p> <p style="margin-right: 50px;">$(I-0^x 1,5,6)$</p> <p style="margin-right: 50px;">$(I-3^x 1,5,6)$</p> <p style="margin-right: 50px;">$(P-0^x 1,2,3,4,5)$</p> <p style="margin-right: 50px;">10 4 -</p>	1 - - - 1 8 0	10 - - - 10 5 9	10 4 - 4 6 3 - - - 5 -	10 4 - 4 6 3 - - - 5 -	10 4 - 4 6 3 - - - 5 -	4 2 5	7 5 8	7 7 9 6	4 4 6 3	7 7 9 6	4 4 6 3	10 - 10 3 11	1 - 1 6 2 -	<p style="text-align: center;">3-2</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="padding: 2px;">5 - - - 5 2 3</td></tr> <tr><td style="padding: 2px;">2 - - - 2 11 0</td></tr> </table> <p style="text-align: center;">5-10</p> <p style="margin-right: 50px;">$(P-3^y 7,11,12)$</p> <p style="margin-right: 50px;">$(P-0^y 7,11,12)$</p> <p>$(I-0^y 8,9,10)$</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="padding: 2px;">7 1 0</td></tr> <tr><td style="padding: 2px;">10 4 3</td></tr> </table> <p style="text-align: right;">3-5</p> <p style="margin-right: 50px;">6-27</p> <p>$(I-3^y 8,9,10)$</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="padding: 2px;">4 4 10 11</td></tr> <tr><td style="padding: 2px;">1 1 7 8</td></tr> <tr><td style="padding: 2px;">4 4 10 11</td></tr> <tr><td style="padding: 2px;">1 1 7 8</td></tr> </table> <p style="text-align: right;">3-5 $(P-3^y 8,9,10)$</p> <p style="text-align: right;">6-27 $(P-0^y 8,9,10)$</p> <p>$(I-0^y 7,11,12)$</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="padding: 2px;">6 - 6 9 8 -</td></tr> <tr><td style="padding: 2px;">9 - 9 0 11 -</td></tr> </table> <p style="text-align: right;">3-2</p> <p style="margin-right: 50px;">$(I-3^y 7,11,12)$</p> <p style="margin-right: 50px;">6 3 -</p> <p style="margin-right: 50px;">5 -</p> <p style="margin-right: 50px;">5-5</p>	5 - - - 5 2 3	2 - - - 2 11 0	7 1 0	10 4 3	4 4 10 11	1 1 7 8	4 4 10 11	1 1 7 8	6 - 6 9 8 -	9 - 9 0 11 -	<p style="text-align: center;">6-5</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="padding: 2px;">10 - 5 9</td></tr> <tr><td style="padding: 2px;">4 ⑥ 3 -</td></tr> <tr><td style="padding: 2px;">7 ⑨ 6 -</td></tr> </table> <p style="text-align: center;">5-10</p> <p style="margin-right: 50px;">$(P-0^x 1,5,6)$</p> <p style="margin-right: 50px;">$(P-0^x 2,3,4)$</p> <p style="margin-right: 50px;">$(P-3^x 2,3,4)$</p> <p style="margin-right: 50px;">3-1</p> <p style="margin-right: 50px;">6-14</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="padding: 2px;">5 4 4 3</td></tr> <tr><td style="padding: 2px;">5 4 4 3</td></tr> <tr><td style="padding: 2px;">5 4 4 3</td></tr> <tr><td style="padding: 2px;">2 1 1 0</td></tr> <tr><td style="padding: 2px;">2 1 1 0</td></tr> </table> <p style="text-align: center;">6-1</p> <p style="margin-right: 50px;">$(P-3^y 7,8,12)$</p> <p style="margin-right: 50px;">$(P-0^x 7,8,12)$</p> <p style="text-align: center;">3-4 $(I-0^y 7,8,12)$</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="padding: 2px;">10 - 3 11</td></tr> <tr><td style="padding: 2px;">1 - 6 2</td></tr> </table> <p style="text-align: right;">6-14</p> <p style="margin-right: 50px;">$(I-0^x 1,5,6)$</p> <p style="margin-right: 50px;">$(I-3^x 1,5,6)$</p> <p style="margin-right: 50px;">6 7</p> <p style="margin-right: 50px;">9 10</p> <p style="margin-right: 50px;">$(I-3^y 7,8,12)$</p> <p style="text-align: center;">5-218</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="padding: 2px;">10 5 9</td></tr> <tr><td style="padding: 2px;">⑥ 3</td></tr> <tr><td style="padding: 2px;">① 8 0</td></tr> <tr><td style="padding: 2px;">⑨ 6</td></tr> </table> <p style="margin-right: 50px;">$(P-0^x 1,5,6)$</p> <p style="margin-right: 50px;">$(P-0^x 3,4)$</p> <p style="margin-right: 50px;">8-26</p> <p style="margin-right: 50px;">$(P-3^x 1,5,6)$</p> <p style="margin-right: 50px;">$(P-3^x 3,4)$</p> <p style="text-align: center;">3-3 $(P-0^y 9,10,11)$</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="padding: 2px;">7 8 11</td></tr> <tr><td style="padding: 2px;">10 11 2</td></tr> <tr><td style="padding: 2px;">7 9 11</td></tr> <tr><td style="padding: 2px;">10 11 2</td></tr> </table> <p style="text-align: right;">3-3 $(P-3^y 9,10,11)$</p> <p style="margin-right: 50px;">$(I-0^x 2,3,4)$</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="padding: 2px;">4 2 5</td></tr> <tr><td style="padding: 2px;">7 5 8</td></tr> </table> <p style="text-align: right;">3-2</p> <p style="margin-right: 50px;">$(I-3^x 2,3,4)$</p> <p style="margin-right: 50px;">5-16</p> <p style="margin-right: 50px;">5-10</p>	10 - 5 9	4 ⑥ 3 -	7 ⑨ 6 -	5 4 4 3	5 4 4 3	5 4 4 3	2 1 1 0	2 1 1 0	10 - 3 11	1 - 6 2	10 5 9	⑥ 3	① 8 0	⑨ 6	7 8 11	10 11 2	7 9 11	10 11 2	4 2 5	7 5 8
1 - - - 1 8 0																																														
10 - - - 10 5 9																																														
10 4 - 4 6 3 - - - 5 -																																														
10 4 - 4 6 3 - - - 5 -																																														
10 4 - 4 6 3 - - - 5 -																																														
4 2 5																																														
7 5 8																																														
7 7 9 6																																														
4 4 6 3																																														
7 7 9 6																																														
4 4 6 3																																														
10 - 10 3 11																																														
1 - 1 6 2 -																																														
5 - - - 5 2 3																																														
2 - - - 2 11 0																																														
7 1 0																																														
10 4 3																																														
4 4 10 11																																														
1 1 7 8																																														
4 4 10 11																																														
1 1 7 8																																														
6 - 6 9 8 -																																														
9 - 9 0 11 -																																														
10 - 5 9																																														
4 ⑥ 3 -																																														
7 ⑨ 6 -																																														
5 4 4 3																																														
5 4 4 3																																														
5 4 4 3																																														
2 1 1 0																																														
2 1 1 0																																														
10 - 3 11																																														
1 - 6 2																																														
10 5 9																																														
⑥ 3																																														
① 8 0																																														
⑨ 6																																														
7 8 11																																														
10 11 2																																														
7 9 11																																														
10 11 2																																														
4 2 5																																														
7 5 8																																														

Schoenberg op. 31, Variation I
 PC set and 12-tone row analysis (pg. 2)

	61	62	63	6-1	3-4
Pic					
Fl 1.			0 - 1 2 (R-0 ^x 12,8,7)		9 5 10 - (R-0 ^x 6,5,1)
Ob 1.	(I-3 ^y 7,11,12) 9 - - - 9 0 11 -	3-4 10 - - 10 3 2 - (I-0 ^x 1,5,6)	3 - 4 5 (R-3 ^y 12,8,7)		9 5 10 - (R-3 ^y 6,5,1)
2.	(I-0 ^y 7,11,12) 6 - - - 6 9 8 -	10 - - 10 3 2 - (I-0 ^x 1,5,6)	2 (11) 10 (R-3 ^y 11,10,9)		0 8 1 - (R-3 ^y 6,5,1)
EH			6-5		6-14 (69) 7 (R-3 ^y 4,3,2)
Es Kl					(36) 4 (R-0 ^x 4,3,2)
Kl 1.2.		1 - - - 1 6 2 - (I-3 ^x 1,5,6)	11 - (8) 7 (R-0 ^y 11,10,9)		(36) 4 (R-0 ^x 4,3,2)
Bs Kl	(P-0 6,7,8,9) 9 2 - - - 1 - 7 -	(P-0 ^y 10,11,12) 8 11 - - 0	11 - (8) 7		(69) 7 3-2 (R-3 ^x 4,3,2)
Fg 1.2.3.	9 2 - - - 1 - 7 -	8 11 - - 0	3-3		
KFg	9 2 - - - 1 - 7 -	8 11 - - 0	3-3		
Hr 1.	7 7 8 - (P-3 6,7,12) 5 - - - 5 - 2 3	4-16 4 6 3 - 3-2 (P-0 ^x 2,3,4) (RI-0 ^y 12,8,7)	8 - 7 6		11 3 (R-10 ^x 4,5,1)
2.	10 10 11 - (P-0 6,7,12) 2 - - - 2 - 11 0	3-2 7 9 6 - (R-3 ^x 2,3,4) (RI-3 ^y 12,8,7)	11 - 10 9		2 6 (RI-3 ^x 4,5,1)
Trp 1.		5-10	1 - 9 0 5-217 (P-3 ^x 1,5,6)		
2.		(P-3 11,12) 2 3	10 - 5 9 6-5 (P-0 ^x 1,5,6)		
Hrf		(P-0 11,12) 11 0	4 6 3		
		4-3	7 9 6		
Cel			5-10 (R-0 ^x 12,8,7) 30 31 41 52 (R-3 ^y 12,8,7)		11 3
			6-1		2 6
Gg 1.	(I-3 ^y 8,9,10) 10 10 4 3	5-10 7 7 5 8 (I-3 ^x 2,3,4)			9 5 10 - (R-0 ^x 6,5,1)
2.	(I-0 ^y 8,9,10) 7 7 1 0 3-5	4 4 2 5 3-2 (I-0 ^x 2,3,4)			0 8 1 - (R-3 ^x 6,5,1)
	10 10 4 3 6-27	7 7 5 8			
	7 7 1 0	4 4 2 5			
Br	4 3 0 (I-3 ^y 9,10,11) (P-0 ^x 8,9,10) 1 1 7 8		7 9 6		5 (RI-0 ^x 4,3,2)
Vcl	1 0 9 (I-0 ^y 9,10,11) (P-3 ^x 8,9,10) 4 4 10 11		4 6 3		8 (RI-3 ^x 4,3,2)
Kbs	(P-0 6,7,8,9) 9 2 - - 1 - 7 -	6-27 (P-0 ^y 10,11,12) 8 11 - 0			
	4-16	3-3			

Schoenberg op. 31, Variation I
 PC set and 12-tone row analysis (pg.3)

	64	65	66
Pic	(R-6 ^{4,3,2}) 9 0 10 (R-9 ^{4,3,2}) 0 3 1	5-10	6-27
F1 1.		4 4 2 5 (I-0 ^{4,2,3,4})	7 7 1 0 (I-0 ^{4,2,3,4})
Ob 1.		7 7 5 8 (I-3 ^{4,2,3,4})	10 10 4 3 (I-3 ^{4,2,3,4})
2.	6-23		
EH (R-9 ^{4,3,2})	8 5 4	6 2 7 -	
Es Kl (R-6 ^{4,3,2})	5 2 1 (3-3)	3 11 4 -	
Kl 1.	5 2 1	3 11 4 (3-4 (R-6 ^{4,3,2}))	
2.	8 5 4	6 2 7 - (R-9 ^{4,3,2})	
Bs Kl	5-16	6-14	6-212
Fg 1.2.3.		4 - - 6 9 - - - 10 -	4 - - - 3 - 8 - (R-I-9 ^{12,11,10,9,8,6})
KFg		4 - - 6 9 - - - 10 -	4 - - - 3 - 8 -
Hr 1.	10		4 - - - 3 - 8 -
2.	1		
Trp 1.	(RI-11 ^{12,8,7}) 11 - 10 9 (RI-37 ^{12,8,7}) 2 - 1 0	(RI-37 ^{6,5,1}) 2 6 1 - (RI-67 ^{6,5,1}) 5 9 4 -	
2.	9-4	(P-0 ^{4,5,6}) 10 - 19 - 5 9 (P-3 ^{4,5,6}) 1 - 1 - 8 0	6-240
Hrf	10	4 4 2 5 7 7 5 8	2 - 2 11 0 - (P-0 ^{4,5,6}) 5 - 5 2 3 - (P-3 ^{4,5,6})
	(RI-6 ^{4,3,2}) 11 8 10 (RI-3 ^{4,3,2}) 8 5 7	3-4	7 7 1 0 10 10 4 3
			1 7 8 (P-0 ^{4,5,6}) 4 10 11 (P-3 ^{4,5,6})
Cel			
Gz 1.	6-1	9 0 10 0 3 1 (3-2)	3-2
2.	6-1	9 0 10 0 3 1	9-2
Br	2 4	0 3 4 - (RI-3 ^{4,3,2})	
Vcl	5 7	3 6 7 - (RI-6 ^{4,3,2})	11 8 10 8 5 7
	9-3 5-16		
Kbs		4 - 6 9 -	10 - 4 - 3 - 8 -
			5-10
			6-212

Schoenberg on. 31, Variation I
 PC set and 12-tone row analysis (ng. 4)

67 5-10 68 69

Pic (P-0^{2,3,4}) 10 0 9 - 6-211 (7) 2, (P-6^{7,8,9,10}) (1) 7 (P-0^{7,8,9})
 Fl 1 (P-9^{7,8,9,10}) 1 (3) 0 - 3-2 5-9 (10) 5-3-5 (P-9^{7,8,9,10}) (4) 10 (P-3^{7,8,9})
 Ob 1. (P-6^{7,11,12}) 4 - 11 2 3-7 5-34 8 - 4 6 (P-6^{7,11,12})
 2. (P-9^{7,11,12}) 7 - 2 6 11 - 8 9 3-2 (P-9^{7,11,12})

EH
 EsKl (P-0^{2,3,4}) (4) 6 3 3-2 (7 8) (P-0^{7,9,10})
 Kl 1. (P-3^{2,3,4}) (7) 9 6 (10 11) (P-3^{7,9,10})
 BsKl
 Fg 1.2.3. 5-5
 KFG
 Hr 1.2. (I-3^{7,9,10}) 4 3 (I-3^{2,3,4}) 4 5 3-2 (I-3^{7,9,10}) 1 0 -
 3.4. (I-6^{7,9,10}) 7 6 4-3 (I-0^{2,3,4}) 4 3 -
 Trp 1. (I-3^{2,3,4}) 6 5 8 3-2 (I-3^{7,9,10}) 4 5 3-2 (I-3^{2,3,4}) 1 0 -
 2. (I-6^{2,3,4}) 10 8 11 5-10 3-4 7-3 4-1 (I-3^{7,9,10}) 4 3 -
 Pos 1.2. 5-212 (I-3^{1,5,6}) 1 - - 6 6 2 9 10 10 0 11 (I-3^{7,8,11,12}) 9 - - 10 (P-0^{10,9,5,2})
 3.4. (I-0^{1,5,6}) 10 - - 3 3 11 6 7 7 9 8 (I-3^{7,8,11,12}) 3 6 - (R-0^{12,11,8,7})

Hrf 10 7 3-10
 1 10 4-17
 4 8
 7 11

Cel 4-28 (1) 9 (P-0^{2,3,4}) (4) 6 3 1 7 (P-0^{7,8,9})
 (3) 0 (P-3^{2,3,4}) (7) 9 6 4 10 (P-3^{7,8,9})
 (4) 11
 (7) 2
 (1) 9
 (3) 0

Xyl 7 11 4-28 (P-0^{1,5,6}) 10 10 10 5 9 9 2 1 11 0 (P-0^{7,8,11,12})
 (P-3^{1,5,6}) 1 1 1 8 0 0 5 5 4 2 3 (P-3^{7,8,11,12})
 Gg 1. 3-4 7 8 4-1
 - 2. (I-3^{1,5,6}) 1 - 6 2 (I-6^{7,8,11,12}) 0 1 1 1 3 2 4-1 7-1
 Br (I-6^{1,5,6}) 4 - 9 5 9 10 10 10 0 11 -7-1 (I-3^{7,8,11,12})
 Val. 5-5
 Cho 5-5 (P-0^{10,9,5,2})

Schoenberg
 Variations for Orchestra, Op. 31
 Theme
 (harp omitted)

34

P₁₀

39 RI₇

43

5-5 in both parts

4-16 in both (= 8-9)

4-7 in accomp. | 3-2 in theme (7-4 in appoggiato)

3-2 in both = 6-2/3: (7, 8, 10, 11, 12)

4-28 invariants

4-16 in accomp.

5-5 in accomp. and theme (here accomp. has same pitch as first phrase of theme)

46

49

13

17

22

46

49

13

17

22

Accomp.
 RI₉

Brackets delimit fixed dyads among set forms:
 B♭ - E and C♯ - G

notice also: correspondence of number of attacks within groups, as given in 9 pc set reduction.

Schoenberg op. 31, Variation I
 PC set and 12-tone row analysis (pg. 5)

	70	71	72
Fl 1.2.3.	(I-6 1,4,7, 8,12) 4 -	4 5 0 1 2 -	3 - - 5 0 - - 1 (R-3 12,7, 6,1)
Cl 1.			7 5 - 3-3-7 (I-3 2,5,10)
EH	5-3	5-3	10 11 1-3-2 (P-6 2,5,9)
Kl 1. (R-3 12,8,7, 6,1)	3 - 4 5 0 - - 1	7 - 6 11 11 10 10 9 (P-9 1,6,7,8,12)	
BsKl		0 - 11 - 8 -	7 - 1 - 2 -
Fg 1.2.3.		0 - 11 - 8 -	7 - 1 - 2 - 6-5-(P-0 12,11,10,9,8,7)
KFg (R-T 12,8,7,6,1) 5-3		0 - 11 - 8 -	7 - 1 - 2 -
Hr 1. (R-3 12,8,7,6,1) 8-19	8 - 7 6 11 - - 10	(I-3 ^x 4,3) 8	0 (R-0 12,7,6) 8 - - 6 11 -
2. (R-10 10) 5,2	0 3 4-3-3	5	4 8 - - 6 11 -
Trp 1. (R-3 9) 5,2	10 8 7 -3-2	1 2 4 -3-2 (P-9 2,5,9)	
2. 7-3	2 9 -4-26	3 5 (P-9 3,4,10,11)	
3. (R-3 3,4,10,11)	11 6 (R-3 3,4,10,11)	0 8 (P-9 3,4,10,11)	
Pos 1.2. 9 5 -4-19 (R-I-0 3,4,11,9)		3-2 7-3 3-3	
3.4. 1 2			
Hrf			10 11 1 -3-2 (P-6 2,5,9)
			0 5 - (P-6 3,4,10,11)
			9 2 -
Gg 1. (I-3 1,4,7,8,12) 5-3		1 - - 2 9 10 - 11	
2.			
Vcl (I-6 2,3,4,5 9,10,11) 7-3	10 9 6 -3-3 (P-6 1,6,7,8, 12)	4 - 3 8 - 7 7 6	11 10 6 7 (R-3 2,3,4,5)
	11 3		2 4 8 9 -
	8 7		
Kbs (R-0 12,11,10,9,8,7)	0 - 11 - 8 -	7 - 1 - 2 - 6-5	9 7 0 1 8

Schoenberg op. 31, Variation I
 PC set and 12-tone row analysis (pg. 6)

	73	74	75
Pic	4-11		3-4
Fl	1 - - 2 9 - - 11 (I-3 4,6,7,12)		10 - 3 11 6 7 7 8 3-1 (I-0 1,2,6,7,8,12)
Ob 1.2.		6-2 10 4 6 - 1 8 - 8 5 1 - 2	1 - 6 2 9 10 10 11 6-1 (I-3 1,2,6,7,8,12)
EsKl		8-3 3 9 8 0 10 11 - 7 6	7 5 8 - 4 3 0 (I-3 2,3,4 9,10,11)
Kl 1.	6-5	3 9 8 0 10 11 - 7 6	4 (2) 5 - 1 0 9 (I-0 2,3,4 9,10,11)
EsKl			5-10 3-3
Fg 1.2.3.	(R-0 6,5,4) 3,2,1	9 5 - - 3 6 - 4 10 -	(P-0 6,5,4,3,2,1)
KFg		9 5 - - 3 6 - 4 10 -	
Hr 1.	11 10 4 - - 3 8 - - 6 (P-6 1,4,7,12)		
2.	11 10 4 - - 3 8 - - 6 1 2 10 10 5 6 0 10 11 7-6		
Tr 1.		11 11 8 4 9 - 7 9 - 4 3 6-16	3-4 6-14
2.			(P-6 1,5,6,7,8,12) 4 - 4 11 3 8
3.			(P-3 1,5,6,7,8,12) 1 - 1 8 0 5
Pos 1.2.		7 - 7 0 3 - - 6 1 - - 8 5 - 2 8-6	
Hrf			(I-0 ¹) 10 6 7 7 8 3-1 (I-0 7,8,12)
			(I-3 ¹) 1 9 10 10 11 6-1 (I-3 7,8,12)
			(I-0 ²) 4
			(I-3 ²) 7 7-26 9-10
Cel			(I-0 1,5) 10 4 (I-0 8)
			(I-3 3,4) 6 (I-3 9,10,11)
			(I-0 3,4) 2 5 (I-0 9,10,11)
			(I-3 3,4) 5 8 (I-3)
			(I-0 3,4) 2 5 6 7 7 8 (I-0 7,8,12)
Glsp			
Gg 1.		8-6 10 - - 5 2 - - 11 4 - - 9 0 - - 3	
2.	5 6 4 3 - 10 - - 5 2 - - 11 4 - - 9 0 - - 3		↑ ↑ 4-17 4-2
	7 8 10 0 - 10 - - 5 2 - - 11 4 - - 9 0 - - 3		
Vcl	5 4 10 0 1 2		
	2 - 9 11 7 5	6-5	
Kbs	(R-0 6,5,4,3,2,1) 5 - 3 6 - 4 10 -		(P-3 2,3,4) 7 9 6 3-2
			(P-6 2,3,4) 10 0 9 5-10

Schoenberg op. 31, Variation I
 PC set and 12-tone row analysis (pg. 7)

	76	77	78
Pic			
11 1.2.3.	$\begin{matrix} 2 & 2 & 1 & 0 \\ 5 & 5 & 4 & 3 \end{matrix}$	$\begin{matrix} 5 & 9 & - & 4 \\ 8 & 0 & - & 7 \end{matrix}$	
Ob 1.2.	$\begin{matrix} 3 & 6 & 7 \\ 11 & 8 & 10 \end{matrix}$		
3.			
EH			
Es Kl	$\begin{matrix} 6 & 9 & 10 \\ 2 & 11 & 1 \end{matrix}$		
Kl 1.2.3.			
Bs Kl			
Fg 1.2.3.			
KFg			
Hr 1.2.3.			
Trp 1.	$\begin{matrix} 7 & 7 & 6 \\ 4 & 4 & 3 \end{matrix}$	$\begin{matrix} 9 & - & 10 & 11 \\ 0 & - & 1 & 2 \end{matrix}$	$\begin{matrix} 6 & 2 & 2 & 7 \\ 9 & 5 & 5 & 10 \end{matrix}$
2.			
Pos 3.			
4.			
Hrf	$\begin{matrix} 6 & 9 & 10 \\ 3 & 6 & 7 \end{matrix}$	$\begin{matrix} 2 & 11 & 1 \\ 11 & 8 & 10 \end{matrix}$	
Cel	$\begin{matrix} 2 & 2 & 1 & 0 \\ 5 & 5 & 4 & 3 \end{matrix}$	$\begin{matrix} 5 & 9 & - & 4 \\ 8 & 0 & - & 7 \end{matrix}$	
Glsp	$\begin{matrix} 3 & 6 & 7 \\ 11 & 8 & 10 \end{matrix}$		
Gg 1.			
2.			
Er			
Vcl	$\begin{matrix} 10 & 5 \\ 0 & 11 \end{matrix}$	$\begin{matrix} 8 & 5 \\ 11 & 8 \end{matrix}$	
Kbs			

76

77

78

$\begin{matrix} 3-1 & 6-1 & 3-4 \\ 6-5 & 6-15 & 3-2 \end{matrix}$

$\begin{matrix} 3-1 & 6-1 \\ 3-1 & 6-1 \end{matrix}$

$\begin{matrix} 5-16 & 3-3 & 6-14 & 3-2 & 5-10 & 6-1 \end{matrix}$

$\begin{matrix} 3-1 & 6-14 & 3-4 \end{matrix}$

$\begin{matrix} 10 & 5 \\ 0 & 11 \end{matrix}$

$\begin{matrix} 10 & 11 \\ 0 & 1 \end{matrix}$

$(RI-6 \ 12,8,7,6,5,1)$
 $(RI-3 \ 12,8,7,6,5,1)$
 $(RI-6 \ 11,10,9,4,3,2)$
 $(RI-3 \ 11,10,9,4,3,2)$
 $(T-9 \ 1,2,3,4,5)$
 $(R-9 \ 12,9,8,7,6,1)$
 $(R-0 \ 12,9,8,7,6,1)$
 $(RI-3 \ 11,10,9,4,3,2)$
 $(RI-6 \ 11,10,9,4,3,2)$
 $(RI-6 \ 12,8,7,5,1)$
 $(RI-3 \ 12,8,7,5,1)$
 $(RI-6 \ 11,10,9,4,3,2)$
 $(R-0^x \ 11,10,9)$
 $(R-3^x \ 11,10,9)$
 $(R-9^x \ 4,3,2)$
 $(R-0^x \ 4,3,2)$

$\begin{matrix} 3 & - & 3 & 4 & 4 & 5 \\ 3 & - & 3 & 4 & 4 & 5 \\ 0 & - & 0 & 1 & 1 & 2 \end{matrix}$

$\begin{matrix} 0 & 8 & 8 & - & - & 1 & - \\ 0 & 8 & 8 & - & - & 1 & - \\ 9 & 5 & 5 & - & - & 10 & - \end{matrix}$

$\begin{matrix} 9 & 5 & 5 & - & - & 10 & - \\ 8 & 3 & & & & 4 & 10 & - \\ 8 & 3 & & & & 4 & 10 & - \\ 8 & 3 & & & & 4 & 10 & - \end{matrix}$

$\begin{matrix} 9 & - & - & 0 & 1 & - \\ 6 & - & - & 9 & 10 & - \\ 5 & - & 5 & - & - & 4 \\ 8 & - & 8 & - & - & 7 \end{matrix}$

$\begin{matrix} 3 & - & 3 & 6 & 4 \\ 3 & - & 3 & 6 & 4 \\ 6 & - & 6 & 9 & 7 \end{matrix}$

$\begin{matrix} 7 & - & 1 & - & - & 11 & 2 & 0 \\ & & & & & 8 & 3 & \\ & & & & & 4 & 10 & - \\ & & & & & 4 & 10 & - \\ & & & & & 4 & 10 & - \end{matrix}$

Schoenberg op. 31, Variation I
 PC set and 12-tone row analysis (pg.8)

f

	79	80	81	3-2	3-5
Pic			(P-3 ^x 2,3,4)	7 - - 9 6	4 10 11 (P-3 ^x 8, 9, 10)
F1 1.2.3.		4 11	8 5 4-2 29 (P-6 1,5,7,11)	7 - - 9 6	4 10 11
Ob 1.		(P-6 2,3, 4,6,8,9,11, 12)	1 2 - 10 9 - (P-0 ^x 2,3,4)	4 - - 6 3	1 7 8 (P-0 ^x 8, 9, 10)
Ob 2.		0 3 -	7 6 - 10 9 -	4 - - 6 3	1 7 8 6-27
EH			0 3 - 4-13	5-10	
Es Kl			0 3 -		
Kl 1.2.3.				7 - - 9 6	4 10 11
BsKl				4 - - 6 3	1 7 8
Fg 1.2.3.				(I-9 10,11,12)	9 - 6 - - - 5 -
KFg					9 - 6 - - - 5 -
Hr 1.3.	1 - (P-9 ^x 1,3,2)	2 - 11 1 - 3-2		10 - 10 - - - 10 5 9	- 3-3 (P-0 ^x 1,5,6)
Hr 2.				7 6 -	
Hr 4.				1 2 -	
Trp			6 - 11 - 2 - 5 - 4-2 29 (I-8 1,5,7,11)		
Pos 1.2.	10 (P-1-6 ^x 4,3,2)	5 2 4 -		1 - 1 - - - 1 8 0	+ 3-3 (P-3 ^x 4,5,6)
Pos 3.	3 - 8 0 0 - - - 7	- 3-4 (P-1-9 4,5,1)			
Pos 4.	6 - 11 3 3 - - - 10	- 5-2 17 (P-1-0 6,5,1)			
Ta			(P-6 1,5,7,11)	4 - 11 - 8 - 5 - 4-2 29	
Pk		10			
Gg 1.	(P-6 1,5 4 - 11 - 8 - 5 -			7	9 6 4 10 11
Gg 2.	7,11) 4 - 11 - 8 - 5 -			7	9 6 4 10 11
B.	(I-8 2,3,4,6, 8, 9, 10)		0 1 - 3 8 -		
B.			10 7 - 9 4 -	(P-3 7, 11, 12)	5 - 5 - - - 5 2 3 - 3-2
Vcl			0 1 - 3 8 -	(P-0 7, 11, 12)	5-10
Vcl			10 7 - 9 4 -		2 - 2 - - - 2 11 0 -
Kbs			4-13		2 - 2 - - - 2 11 0 -
Kbs			4-8		(I-9 10, 11, 12)
Kbs					9 - 6 - - - 5 - 3-3

THEME

Molto moderato (♩ = 88)

P-O

Viol 3/4 *p* *part*

Kbn 4/4 *p*

34 35 36 37 38 39 40 41

U. E. 12196

Viol 42 43 44 45 46 47 48 49

Kbn *pp*

R-O

I. GR
m. Dpf

Viol *pp* *dolce*

Kbn *pp* *dolce*

50 51 52 53 54 55

U. E. 12196

Variation I

Kbn 58 59 60 61

P-O

Kbn 62 63 64

R-I-9

Kbn 65 66 67 68

Kbn 69 70 71 72

R-O

Kbn 73 74 75 76

Kbn 77 78 79 80 81

I-9

rit. attacca

THEME

Measures:

Row:

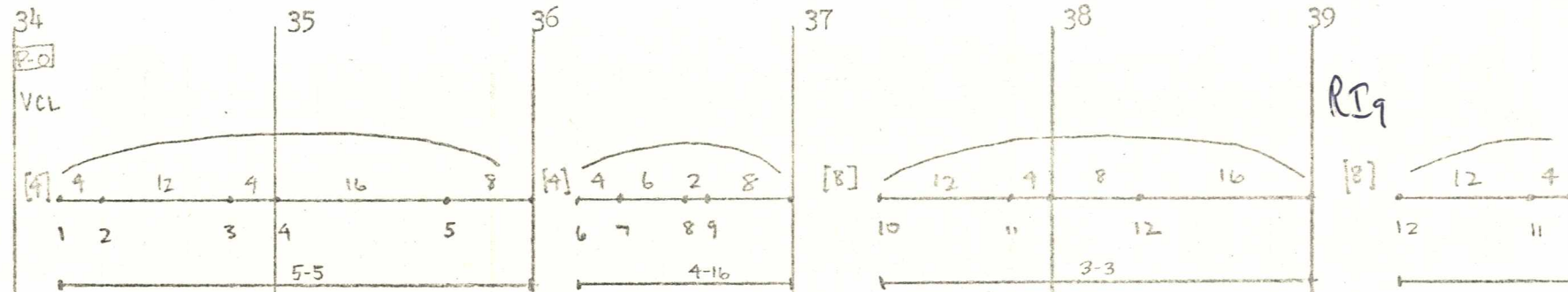
Instrumentation:

Phrasing:

R-durations:

Ordering:

PC sets:



VARIATION I

Measures:

Row:

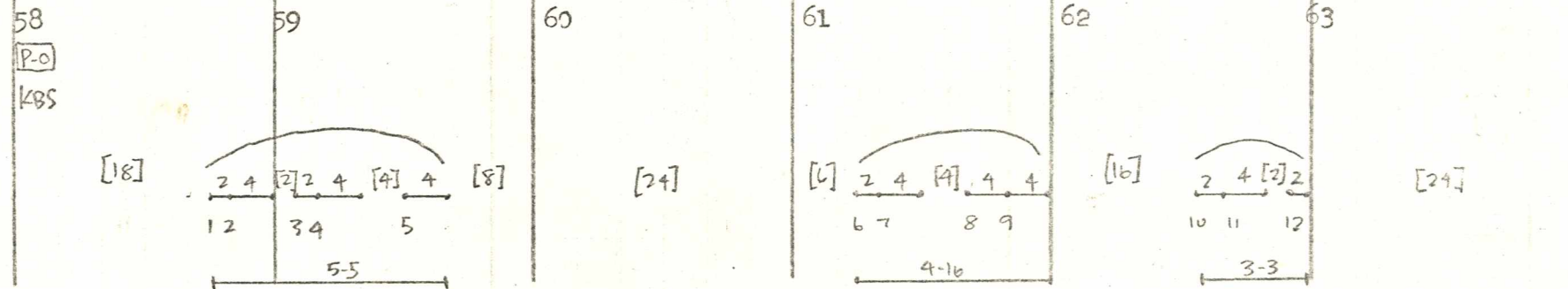
Instrumentation:

Phrasing:

R-Durations:

Orderings:

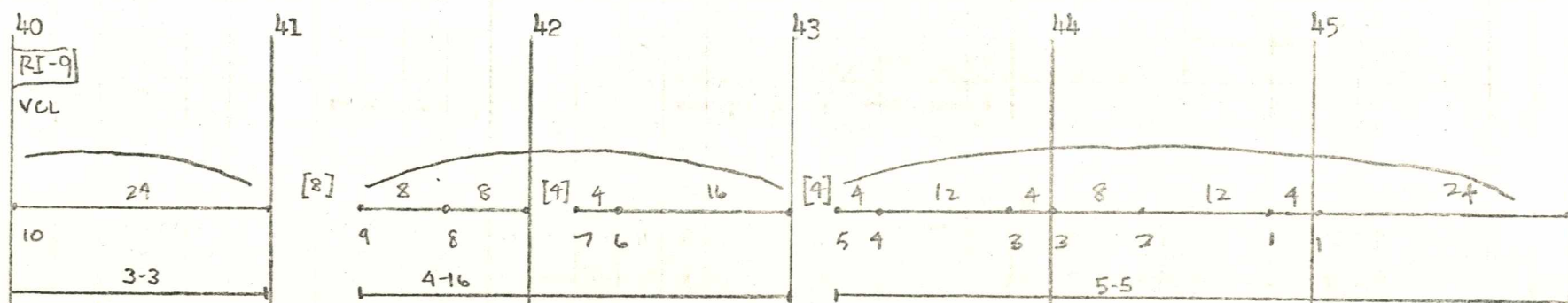
PC sets:



	I-0	I-6	I-8	I-5	I-7	I-11	I-4	I-3	I-9	I-10	I-1	I-2
P-0	10	4	6	5	5	9	2	1	7	8	11	0
P-6	4	10	0	9	11	3	8	7	1	2	5	6
P-4	2	8	10	7	9	1	6	5	11	0	3	4
P-7	5	11	1	10	0	4	0	2	2	2	6	7
P-5	3	0	11	2	10	0	7	2	0	1	4	7
P-1	11	5	7	4	6	10	3	2	2	9	0	1
P-8	6	0	2	11	1	5	10	9	3	4	7	8
P-9	7	1	3	0	2	6	11	10	4	5	8	0
P-3	1	7	9	6	8	0	5	4	10	11	2	3
P-2	0	6	8	5	7	11	4	3	9	10	1	2
P-11	9	3	5	2	4	8	1	0	6	7	10	11
P-10	8	2	4	1	3	7	0	11	5	6	9	10

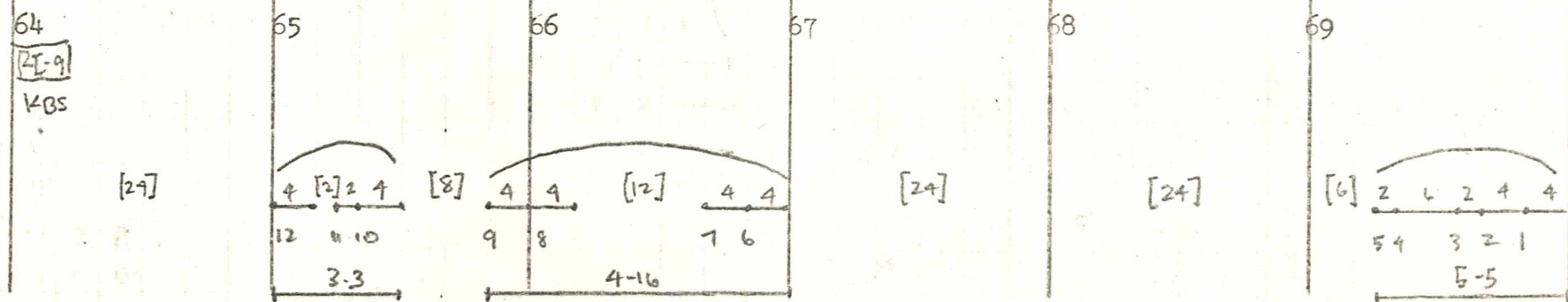
THEME

Measures:
 Row:
 Instrumentation:
 Phrasing:
 R-durations:
 Ordering:
 PC sets:



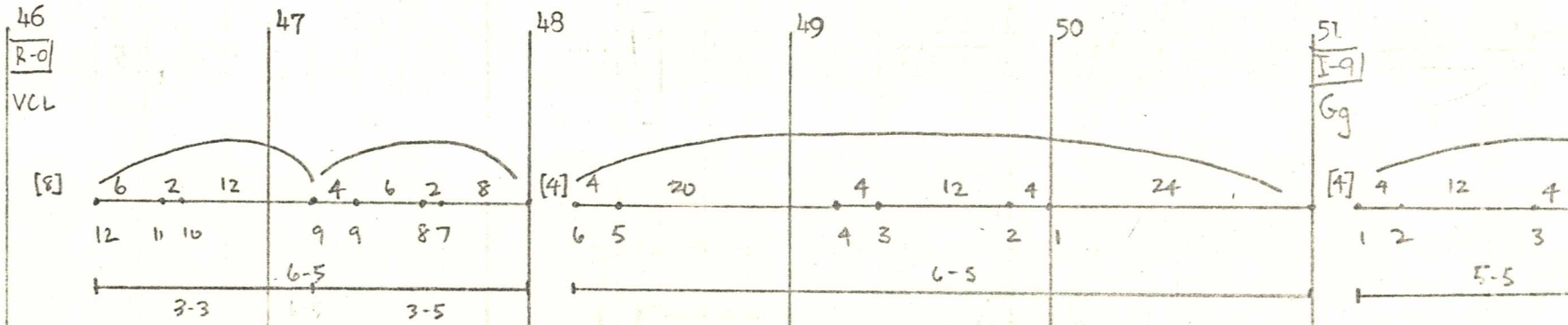
VARIATION I

Measures:
 Row:
 Instrumentation:
 Phrasing:
 R-durations:
 Orderings:
 PC sets:



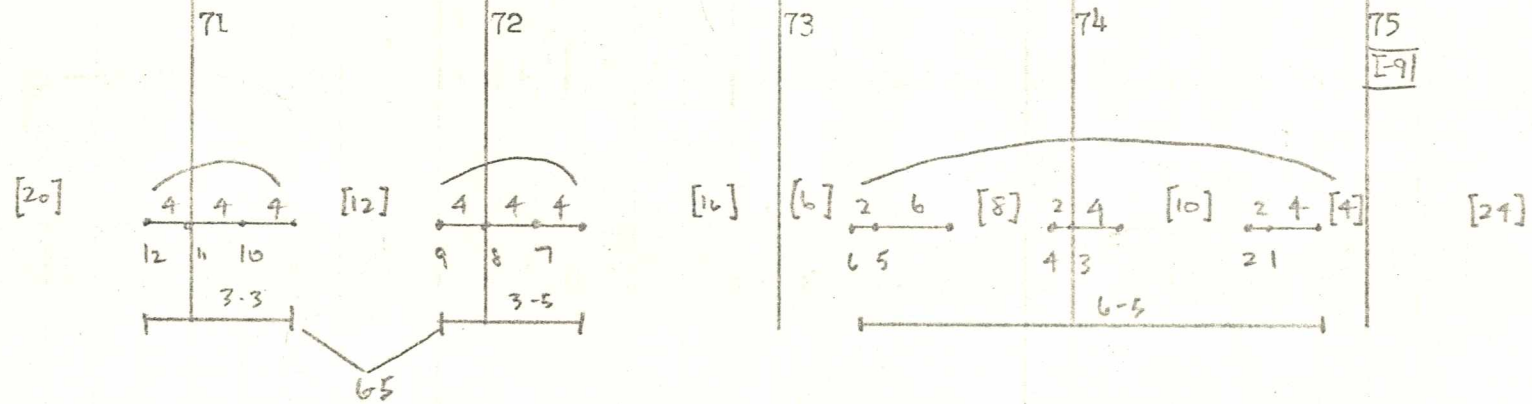
THEME

Measures: 46
 Row: R-0
 Instrumentation: VCL
 Phrasing:
 R-durations: [8]
 Ordering:
 PC sets:



VARIATION I

Measures: 70
 Row: R-0
 Instrumentation: KBS
 Phrasing:
 R-durations: [20]
 Orderings:
 PC sets:



THEME

Measures:

Row:

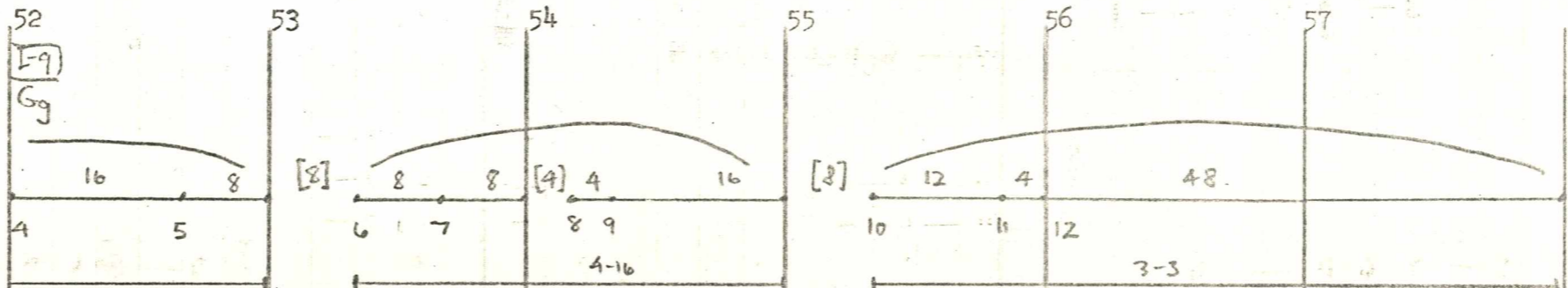
Instrumentation:

Phrasing:

R-durations:

Ordering:

PC sets:



VARIATION I

Measures:

Row:

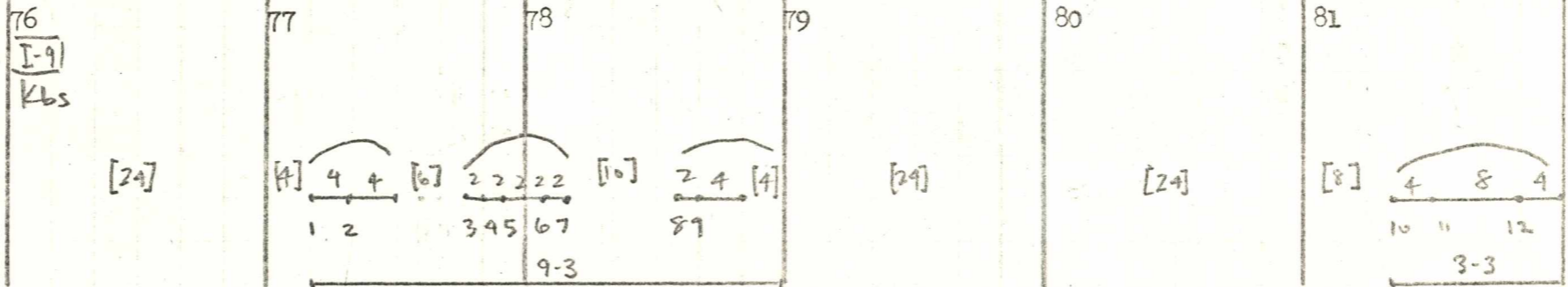
Instrumentation:

Phrasing:

R-durations:

Ordering:

PC sets:



Introduction:

set-form combination determined by fixed dyads 0,6 and 3,9 — Here and in the variations with but few exceptions (Var. IV)

These are invariant (with possible reversal) ^{only} in

P ₀ (R ₀)	I ₀ (RI ₀)
P ₆ (R ₆)	I ₆ (RI ₆)
P ₉ (R ₉)	I ₉ (RI ₉)
P ₃ (R ₃)	I ₃ (RI ₃)

Combinatorial hexachords:

$$P_0^a = I_9^b$$

$$P_6^a = I_3^b$$

$$P_9^a = I_6^b$$

$$P_3^a = I_0^b$$

only P₀ and I₉ are combined in the introduction

These set-forms are combined in various ways (see sep. pages) in the introduction--the only exception being RI₁ in m. 25 (to obtain B-A-CH)

The unique succession P₀ RI₁ in mm. 24 and 25 is based upon

a correspondence of the dyads forming B-A-C-H and common tetrachordal segments between the two forms:

P ₀	0, 6, 8, 5, 7, 11, 4, 3, 9, 10, 1, 2
RI ₁	11, 0, 3, 4, 10, 9, 2, 6, 8, 5, 7, 1

m. 24

Fl.	4	3	9	10	<u>H</u>	<u>C</u>	<u>A</u>	<u>B</u>	3	4	10	9
Vc.	8			7			8			7		
E.H.	6	5					6	5				
Pos.	<u>B</u>			<u>A</u>			<u>C</u>				<u>H</u>	

P₀

RI₁

1-combinatorial (1 value of +)
 Same hexachord as Moses und Aron (6-5)
 and Op. 33b

[422232]
 ↑

Op. 31
 Set table

P →
 I ↓
 R ←
 RI ↑

	B ^b	E	F [#]	E ^b	F [#]	A [#]	D [#]	C [#]	G [#]	G [#]	B [#]	C [#]
B ^b	0	6	8	5	7	11	4	3	9	10	1	2
E [#]	6	0	2	11	1	5	10	9	3	4	7	8
D [#]	4	10	0	9	11	3	8	7	1	2	5	6
F [#]	7	1	3	0	2	6	11	10	4	5	8	9
E ^b	5	11	1	10	0	4	9	8	2	3	6	7
B [#]	1	7	9	6	8	0	5	4	10	11	2	3
F [#]	8	2	4	1	3	7	0	11	5	6	9	10
G [#]	9	3	5	2	4	8	1	0	6	7	10	11
C [#]	3	9	11	8	10	2	7	6	0	1	4	5
C [#]	2	8	10	7	9	1	6	5	11	0	3	4
A [#]	11	5	7	4	6	10	3	2	8	9	0	1
G [#]	10	4	6	3	5	9	2	1	7	8	11	0

Row form numbers:

- P₀: Prime beginning on pc0
- I₉: Inversion beginning on pc9
- R₀: Retrograde of P₀ (not retrograde beginning on pc0)
- RI₉: Retrograde of I₉ (not retrograde inversion beginning on pc9)

This shows more clearly the correspondences among row forms, whether used in combination or in succession.

Op. 31

Style and Idea

Leibowitz

Margaret

Penle

Raper ~~St~~, 92, (93-4) ~~on~~

Fennelly

on introduction

~~Style and Idea~~

~~1911, 1912, 1913, 1914~~

Partitioning of theme 5 4 3

~~Variations on what?~~

~~First performance~~

Dec. 2, 1928, Berlin Philharmonic, Wilhelm Furtwängler - scandal (hiss) at opening (not unusual)

~~Difficulties of individual parts~~

B-A-C-H motifs in the work - Fennelly sees Style & Idea p. 141
no. 24 Trombones

~~First growth to employ full orchestra using 12-tone method - technical problems in combining new forms - no octaves doubling~~

See Style & Idea on separate variations

as distinct from
various doublings

Large orchestra: Orchestration - clarity as goal (quint Style (Idea))

Rhythmic organization of the melodic theme
 () = rest

P₀

	0	6	8	5	7		11	4	3	9		10	1	1	2	
	2	6	2	8	4	(2)	2	3	1	4	(4)	6	2	4	8	(4)

(Note: Brackets and underlines are present in the original image)

62

R_{1g}

	7	8	11		0	6		5	10		2	4	1	1	3	9	9	
	6	2	12	(4)	4	4	(2)	2	8	(2)	2	6	2	4	6	2	12	(4)

(Note: Brackets and underlines are present in the original image)

84

R₀

	2	1	10		9	9	3	4		11	7	5	8	6	0
	3	1	6	2	3	1	4	(2)	2	10	2	6	2	12	(2)

(Note: Brackets and underlines are present in the original image)

56

I_g

	9	3	1	4	2		10	5		6	0		11	8	8	7
	2	6	2	8	4	(4)	4	4	(2)	2	8	(4)	6	2	12	12

(Note: Brackets and underlines are present in the original image)

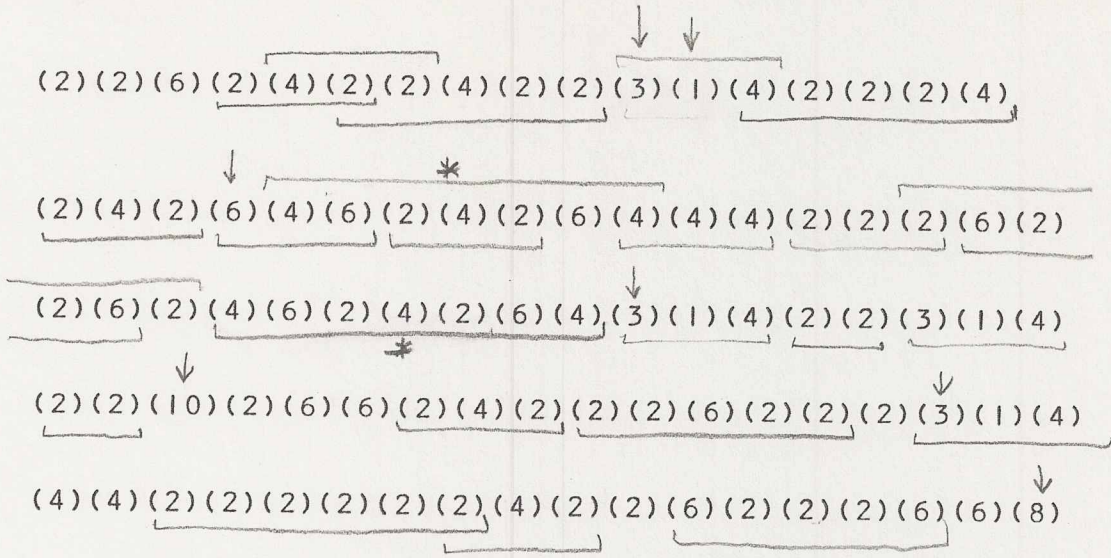
m. 34 m. 41 m. 42 m. 37 - not quite

(Note: A large bracket spans from the first measure to the last measure)

82

Schoenberg, Op.31 Theme

Attack-Release Partition



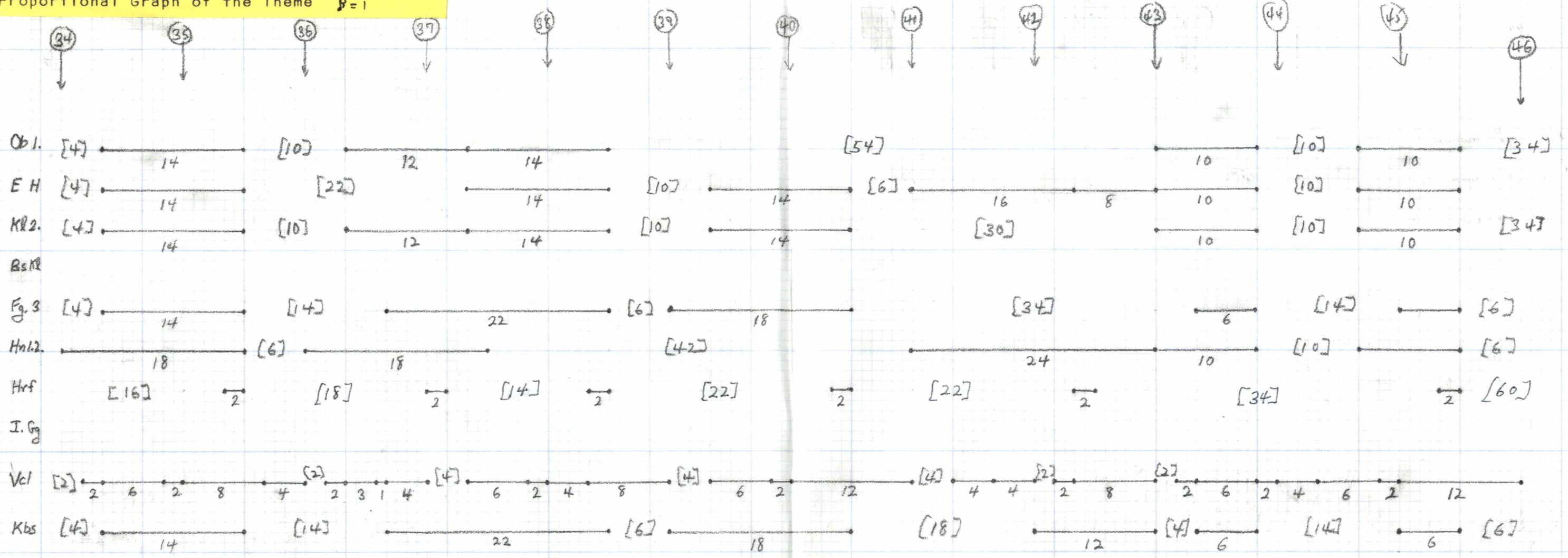
Patterns:

- many repetitions
- many palindromes--with and without an axis of symmetry
- special durations: 1, 3, 8, 10

to be checked



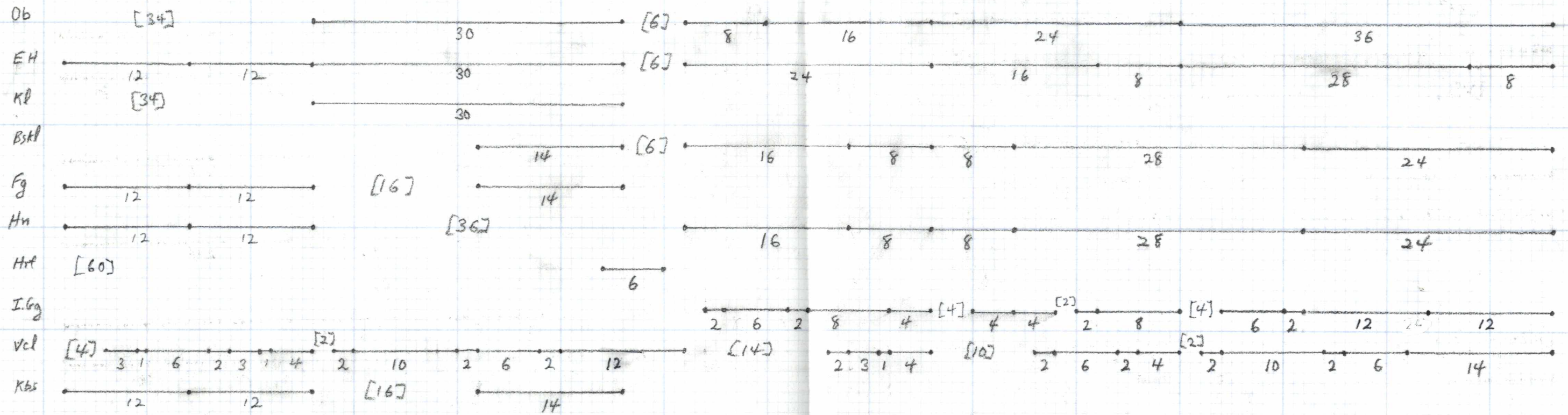
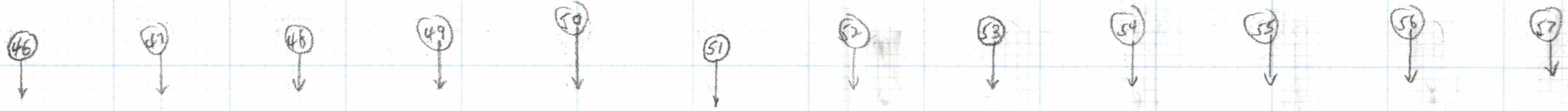
Schoenberg, Variations for Orchestra, Op.31
 Proportional Graph of the Theme $\beta=1$



P₀
I₉

RI₉
R₀

R₀
RI₉



R₀
RI₀

I₀
P₀

P₃ (vcl)