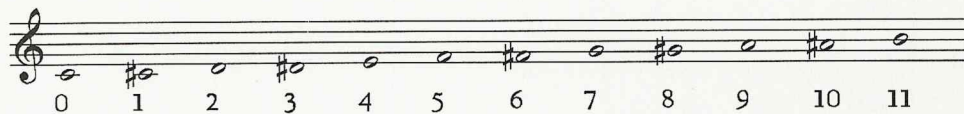
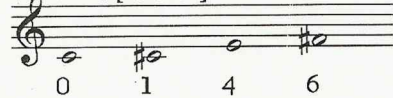


## Integer notation

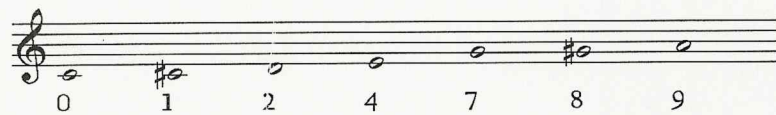


### Examples of Pitch-class Sets

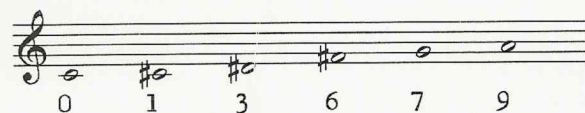
a) 4-z15:[0,1,4,6]



b) 7-20:[0,1,2,4,7,8,9] [433452]



c) 6-30 (octatonic) [224223]



### Comments:

1. As shown at a), the integers from 0 through 11 replace the traditional letter-names or solfege syllables. These associations of pitch and integer are fixed. For example, any of the three normal notational forms of the pitch that sounds C (C, B#, Dbb) is always associated with the integer 1. Thus, each of the twelve integers represents a pitch class.

This numeric representation permits arithmetic to be performed on the pitch-class integers, whereas, for example, it is not possible to perform arithmetic operations on letter-names or solfege syllables!

2. A collection of pitch-class integers, without duplicates, is called a pitch-class set. One such set is shown at b). It consists of the integers 0, 1, 4, and 6. This set has the name 4-z15, which represents a single class of sets, each member of which is related to all the others by transposition or inversion. Thus, we speak of a pitch-class set class. There are exactly 220 pitch-class set classes containing from 2 to 10 pitch classes.

The set-class name has the form  $i_1-(z)i_2$ , where the integer  $i_1$  is the number of pitch classes in the set, the hyphen is a fixed delimiter, the letter z is an optional descriptor, and the integer  $i_2$  locates the set on a list of sets of the same size. Thus, pitch-class set 4-z15 contains four pitch classes, it has a special characteristic designated by the letter z, and it is the 15th set on an ordered list of 4-note sets (tetrachords), of which there are 29 altogether.

Pitch-class set (pc set) 7-20, at c) is a staple item in Messiaen's harmonic vocabulary. As indicated by its set name, it contains seven notes and is the 20th set on the list of 38 septads.

Pc set 6-30, at d) is the 30th set on the list of 50 hexachords. In parentheses I have supplied the additional information that hexachord 6-30 is often associated with the octatonic scale, whose pitch-class set class name is 8-28.



4-18	5-28	4-26
CI	CII	CIII

Vingt Regards VI, Theme d'amour

	G1	G2	G3	G6	G9	G12
4-18	○		■		○	
4-26						■
5-28	○	○	■	○		○

Squo Indices

- .222: G3 (diminished), G12 (dia-tonal)
- .158: G1 (atonal)
- .121: G9 (atonal-tonal)
- .111: G6 (semichroma)
- .076: G2 (whole-tone)

Genus 3 and Genus 12 account for all the sets. Why is 4-26, an octatonic tetrachord, not in Genus 3? Because the genera are not oriented toward scales! Nor does genera membership depend upon the subset (inclusion) relation (cf. Parks). The axiomatic trichordal basis is determinant.



(8va)

7-z36 7-z12 7-z36 7-20 7-20 (T10) 7-20

The opening chords of 'Les Mains de L'Abime' comprise two pairs. Although the first of these pairs consists of sets from different classes (7-z36 and 7-z12), these sets have the same interval content [444342]. When Messiaen discusses this group of three chords in TRCO III (p.188) his notation retains the low D-E dyad in the second chord. The second chord pair, in contrast, consists of a single set class, septad 7-20, a favorite of Messiaen's. Interval content [433452]. Thus, the first chords are equivalent with respect to interval class, while the second are equivalent with respect to pitch class. The bass dyad C-B, has symbolic significance in the work.

Les Mains de L'Abime, outer parts

Soprano Bass

4-23 (dia) 4-3 (octa)

201

202

203

Accord sur dominante appoggiaturé

204

*p expressif*

T10

7-20/7-35

7-20/7-35 (FH)

340

Klar.  
Solo-Geige  
Oboe  
Solo-Bratsche  
Klar. Horn  
Bass  
Solo-Vcl.  
Horn  
pp, Klar.  
Solo-Bratsche  
Bass-Klar.  
Bass-Tuba  
Klar.-Fag.

In diesem Akkord kommen elf verschiedene Töne vor. Aber die zarte Instrumentation und, daß die Dissonanzen weit auseinander liegen, macht, daß dieser Klang sehr weich wirkt. Aber vielleicht noch eins. Die einzelnen Gruppen sind so gesetzt, daß man sie leicht auf frühere Formen zurückführen könnte. Beispielsweise den ersten Einsatz. Ich glaube, das Ohr erwartet hier diese Auflösung:

341

Bar 12: first 'guirlande'.

Boundary pitches of contour groups

7-20:  
[0,6,9,11,2,3,4]

Bar 20: third guirlande

The upper strand of this bi-linear figuration unfolds 7-20, an inverted form of the first 7-20 in bar 3 (T<sub>4</sub>I). The parsing here is also one favored by Messiaen, the saute-mouton, or leapfrog. In French, sheep, not frogs, leap over each other. In the lower strand, the hexachord 6-16, one of the seven hexachords of 7-20, unfolds. It lacks a single pc to become a second form of 7-20 in this melodic 'garland', viz., pc3, or Eb.



"Les Mains de L'Abime" Bar 29: 12-tone row

3 2 2 2 2 2 3  
non-retro contour groups

Involution

"Les Mains de L'Abime" Bar 29: disjunct hexachords

6-z46:[6,7,8,10,0,3] 6-z24:[9,11,1,2,4,5]

Non-retro contour groups: 3 2 2 2 2 3

6-z24/46 interval vector: [2 3 3 3 3 1]  
N.B. ic6 counts twice; thus the non-retro pattern is 2 3 3 3 2.

"Les Mains de L'Abime" Bar 29: Headnotes of contour groups

6-z46:[2,3,4,6,8,11] 6-z24:[5,7,9,10,0,1]

Contour groups and contour in general form yet another programmatic parameter, the mountainous terrain of the Dauphiné (along with register, dynamics, etc.)

"Les Mains de L'Abime" Bar 29: 12-tone row  
Involution

Involution: disjunct hexachords

6-z38:[11,0,1,2,6,7] 6-z6:[3,4,5,8,9,10]

Involution: Headnotes of contour groups

5-20 7-20

Septad 7-20, here in linear form, is  $T_{11}$  of the first 7-20, bar 3, and  $T_1$  of the second form. Since the transposition operators are inverse-related mod 12, the same set-class is held invariant with respect to each of the "original" forms of 7-20. This connection, although remote, is interesting from the standpoint of abstract set structure.

4-8

### Messiaen's Modes of Limited Transposition as Members of Genera

	G1	G2	G3	G4	
9-12				o	Mode 3
8-9	o				Mode 4
8-25		o			Mode 6
8-28			o		Mode 2
6-7	o	o			Mode 5
6-35		o		o	Mode 1

### Squo Indices

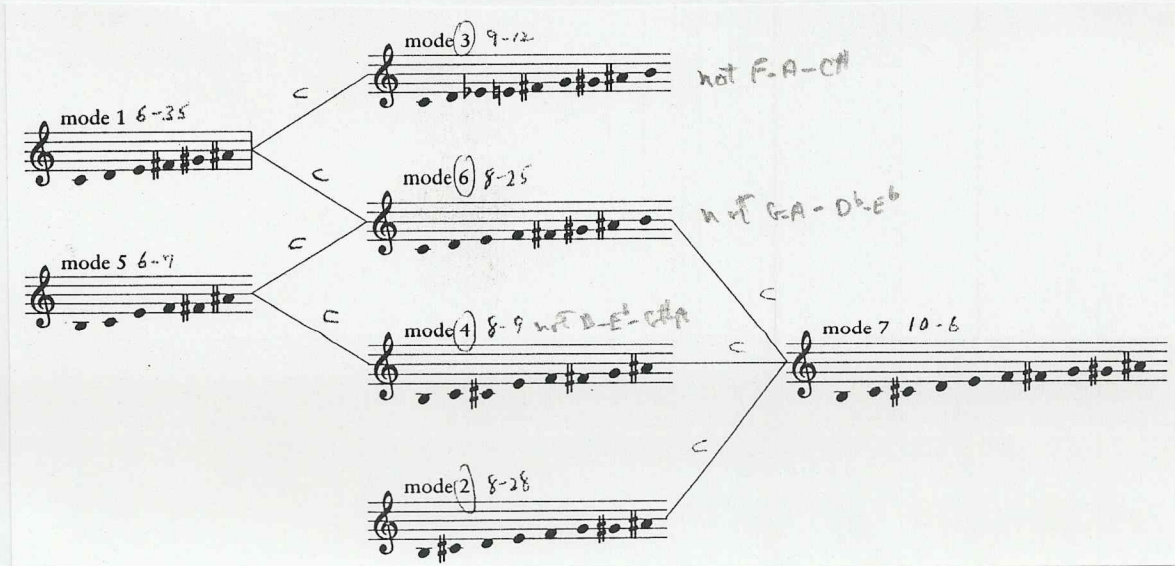
- .190: G4 (augmented)
- .092: G2 (whole-tone)
- .063: G1 (atonal)
- .044: G3 (diminished)

Mode 7 (pc set 10-6) is omitted from the matrix, which will only deal with sets of sizes 3 through 9.

Although Messiaen's modes fit neatly into the first four genera, two of them belong to two genera (hexachords 6-7 and 6-35).

Looking at the modes from the perspective of the genera, Genus 2, dubbed 'whole-tone' claims three of the modes, whereas Genera 1 and 4 each claim two. ~~Only one mode belongs to a single genus, the 'octatonic' mode, pc set 8-28~~ genre contains a single mode, 6-7.

Squo (status quotient) is a statistical measure that shows the relative strength of the genus within the complete array of four genera in this matrix.



Anthony Pople. "Messiaen's Musical Language: an Introduction. In Peter Hill, ed. The Messiaen Companion. London: Faber & Faber, 1994: 15-50



Prelude 2

tr

Mode 2: 5-19:[0,1,3,6,7]

La colombe

mode 2  
8-28 CI (off)

Lent, expressif, d'une sonorité très enveloppée

1-2

4-18 CI

"E-majour"

3-6

6-33 (E-major hexachord)

4-23

expressif

7-9

6-33 (G-major hexachord)

4-3 (b.2)

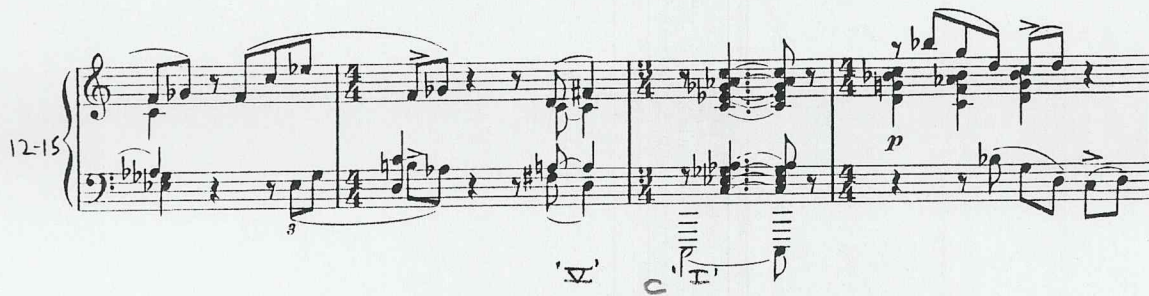
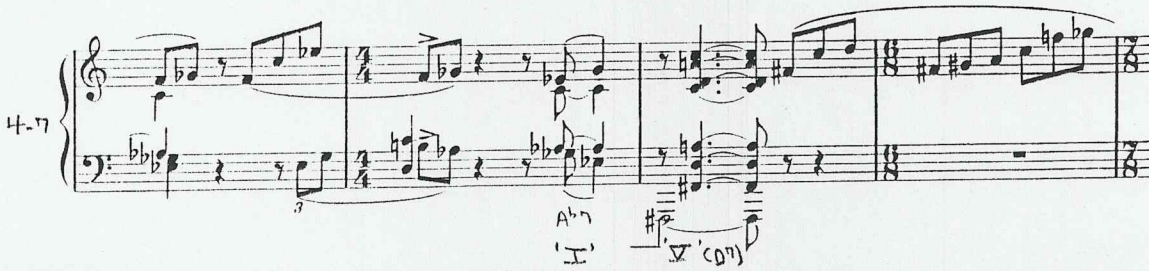
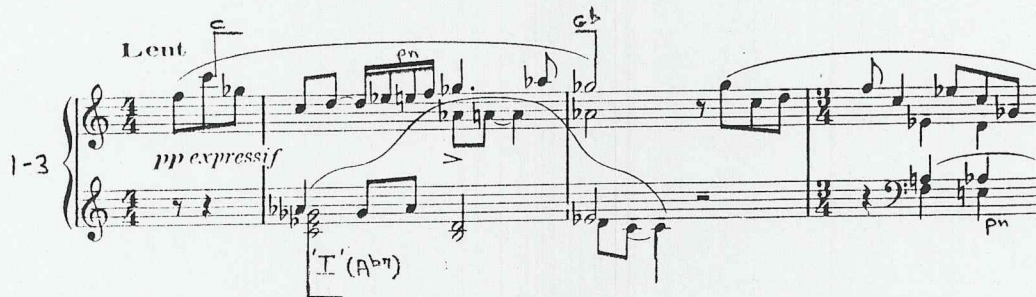
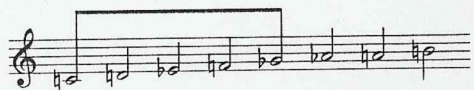
cresc.

CI: 6-249 6-250 4.19 4.27

# Prelude

pour piano

OLIVIER MESSIAEN



### Tonic/dominant analogues

a) 6-3C: common tritone b) voice-leading



### Bass bs 1-6 (6-z13)



Bars 13-14

Not:

b) But:





Bien modéré, en poudrolement harmonieux (comme un oiseau) 5-28

VIOLON

CLARINETTE en SI b

VIOLONCELLE

PIANO

*pp* (*son flûte*)

*p expressif*

*pp legato (très enveloppé de pédale)*

*ppp (vibrato)*

4-8 (comme un oiseau) 4-9 6-26 5-33 (4-21) 8-8

5-219 (5-22/5-218)

**A** Bien modéré, en poudrolement harmonieux (♩=54 environ)

1 2 3 4 5 6 7 8 9 10

von

Clar.

vclle

PIANO

vers la pointe)

(5-28)

4-8 8-5 4-18

4-9 4-18

*glissando* (\*) *gliss.*

11 12 13 14 15 16 17 18 19 20

von

Clar.

vclle

PIANO

**B**

5-13 4-8

21 22 23 24 25 26 27 28 29 | 1 2 3 4 5

Messiaen *Quatuor pour la fin du temps*: the 29 Chords

No.	Name	Normal order	Octa Transformations/Comments
1	7-20:	3 4 5 7 10 11 0	T <sub>0</sub> Messiaen signature
2	7-35:	9 10 0 2 3 5 7	T <sub>0</sub>
3	7-20:	1 2 3 5 8 9 10	T <sub>10</sub>
4	7-35:	7 8 10 0 1 3 5	T <sub>10</sub>
5	7-20:	10 11 0 2 5 6 7	T <sub>9</sub>
6	7-35:	4 5 7 9 10 0 2	T <sub>9</sub>
7	7-20:	5 6 7 9 0 1 2	T <sub>7</sub>
8	7-35:	11 0 2 4 5 7 9	T <sub>7</sub>
9	5-z38:	10 1 4 5 6	
10	5-25:	11 2 4 5 7	CI T <sub>0</sub>
11	5-27:	0 3 5 7 8	T <sub>0</sub>
12	6-34:	4 5 7 9 11 1	nearly wt (mystic chord)
13	5-27:	3 6 8 10 11	T <sub>3</sub>
14	5-35:	1 3 5 8 10	T <sub>0</sub>
15	5-35:	11 1 3 6 8	T <sub>10</sub>
16	5-21:	10 1 2 5 6	
17	6-15:	6 9 10 0 1 2	T <sub>0</sub>
18	6-21:	8 10 0 1 2 4	nearly wt
19	6-14:	9 0 1 2 4 5	
20	6-15:	10 1 2 4 5 6	T <sub>4</sub>
21	6-15:	2 5 6 8 9 10	T <sub>4</sub>
22	6-z23:	3 5 6 8 9 11	CII
23	5-25:	2 4 5 7 10	CI T <sub>9</sub> I
24	4-z29:	1 2 4 8	CI T <sub>0</sub> All-interval
25	5-25:	11 1 2 4 7	CI T <sub>9</sub>
26	4-z29:	10 11 1 5	CI T <sub>9</sub>
27	4-z29:	4 8 10 11	CI T <sub>9</sub> I
28	4-z29:	1 5 7 8	CI T <sub>9</sub>
29	4-z29:	11 3 5 6	CII T <sub>10</sub>

Unordered transformations among sets of the same class are shown as T<sub>i</sub> (transposition) or T<sub>i</sub>I (inversion) where i is the transposition operator. Successive transpositions of sets of the same class are computed with reference to the last chronological form beginning with the initial form T<sub>0</sub>. For example, 6-15 is first transposed up 4 semitones (T<sub>4</sub>) then the new form is transposed at the same interval (T<sub>4</sub>). Note that the transpositions are "unordered."

- CI: 1 2 4 5 7 8 10 11
- CII: 2 3 5 6 8 9 11 0
- CIII: 3 4 6 7 9 10 0 1



Quatuor: The 12 set classes of the 29 chords

	G1	G2	G3	G4	G5	G6	G7	G8	G9	G10	G11	G12	
4-z29	■	○											2
7-20	○	○								■			2
5-21				■				○	○	○			4
5-25	○	○	*				○					■	5
5-27							○			○	○	■	4
7-35											○	■	2
5-z38	○	○	○					○	○	○		■	7
6-14				■	○	○	○	○	○	○	○	*	9
6-15	○	○	○	■	○	○	○	○	○	○		*	11
6-21	○	○	○	■	○	○	○	○	○	○		*	11
6-z23	○	○	*			○	○					■	6
6-34	○	○	○	■		○	○	○	○	○	○	*	11
Total	8	8	6	5	3	5	7	6	6	8	4	9	
Reduced	1			5						1		5	

Squo Indices in Descending Order with Genera

- .216: G4 (augmented)
- .181: G12 (dia-tonal)
- .177: G10 (atonal-tonal)
- .141: G7 (chroma-dia)
- .133: G8 (atonal), G9 (atonal-tonal)
- .125: G11 (dia)
- .121: G3 (diminished)
- .115: G1 (atonal)
- .111: G2 (whole-tone)
- .101: G6 (semichroma)
- .094: G5 (chroma)

Although the 12 set classes engage all 12 genera, only <sup>4</sup>5 genera remain active after judicious application of reduction strategies, primarily derived from the status quotient (squo) calculation, which measures the status of a genus by taking into account the number of representatives of a genus (X), the total set count for the matrix (Y) and the total size of the genus (Z). For a genus Ga:  $Squo(Ga) = ((X / Y) / Z) \cdot 10$ . The constant 10 simply shifts the decimal number one place to the left for greater legibility. Matrix display and calculations are done by computer program.

Notice that the sets vary greatly with respect to number of genera affiliations. Whereas 4-z29, one of the all-interval tetrachords, belongs to only two genera, hexachords 6-15, 6-21, and 6-34 (Scriabin's mystic chord) are very gregarious, avoiding only the pure diatonic genus, G11.

The reduced matrix dramatizes the conflict between Genus 4 (augmented) and Genus 12 (diatonic) in four instances, with Genus 4 winning preference, largely because of its small size and the corresponding value of Z. Because the difference in Squo is so small, I have compensated by assigning the four sets involved in this dilemma to G12 as well, indicating their secondary membership by \*.

Forte, Allen. "Pitch-Class Set Genera and the Origin of Modern Harmonic Species." *Journal of Music Theory* 32:2, Fall 1988: 187-270.

Theme d'accords

a)

4-23                      4-11                      #5

4-26                      4-23                      #5

4-23                      4-8                      4-18                      4-5

b)

Inner voices: 6-30 CII                      Outer voices: 6-30 CI  
4-z29

4-13                      4-18

c)

Inner: 6-30 CII

Outer: 6-30 CI T1I

The Thème d'accords from *Vingt Regards* succinctly illustrates the important role played by horizontal components in Messiaen's music.

At a) the set name of each chord is placed below the vertical, showing the succession 4-23, 4-8, 4-18, and 4-5. A diverse assemblage. The two set names above the lower staff apply to the pair of dyads, thus, 4-26 and 4-23, and the upper staff is labelled in the same way.

At b) are displayed the linear-horizontal components, first the pair of inner voices, then the pair of outer voices. These comprise two forms of the same hexachord, octatonic 6-30, the inner drawn from Collection II, the outer from Collection I, hence a self-enclosed partial octatonic cycle.

At c) these inner voice and outer voice successions are summarized in scalar form. The outer voices are an inverted form of the inner ( $T_1I$ , to be precise). Together, the two hexachords exhaust the total chromatic.

It is likely that Messiaen considered hexachord 6-30 the 'perfect' hexachord, at least in the context of Mode 2.



Vingt Regards, Theme d'accords: Reduced Matrix

	G1	G3	G7	G11
4-5	■			
4-8	■			
4-11			■	
4-13	■	*	○	
4-18	■	*		
4-23				■
4-z29	■			
6-30	■	*	○	

Squo Indices

- .136: G1 (atonal)
- .095: G3 (diminished), G7 (chroma-dia)
- .049: G11 (dia)

The tetrachords of the Theme d'accords are overwhelmingly associated with Genus 1, the Genus of modes 4 and 5, which are not represented here. Tetrachords 4-11 and 4-23 are singletons, each belonging to only one genus, hence of special character. Both are familiar in ordinary diatonic music. Tetrachord 4-23 plays a special role in Messiaen's music, often involved in voice-leading, as in the 29 chords of *Liturgie de cristal*.