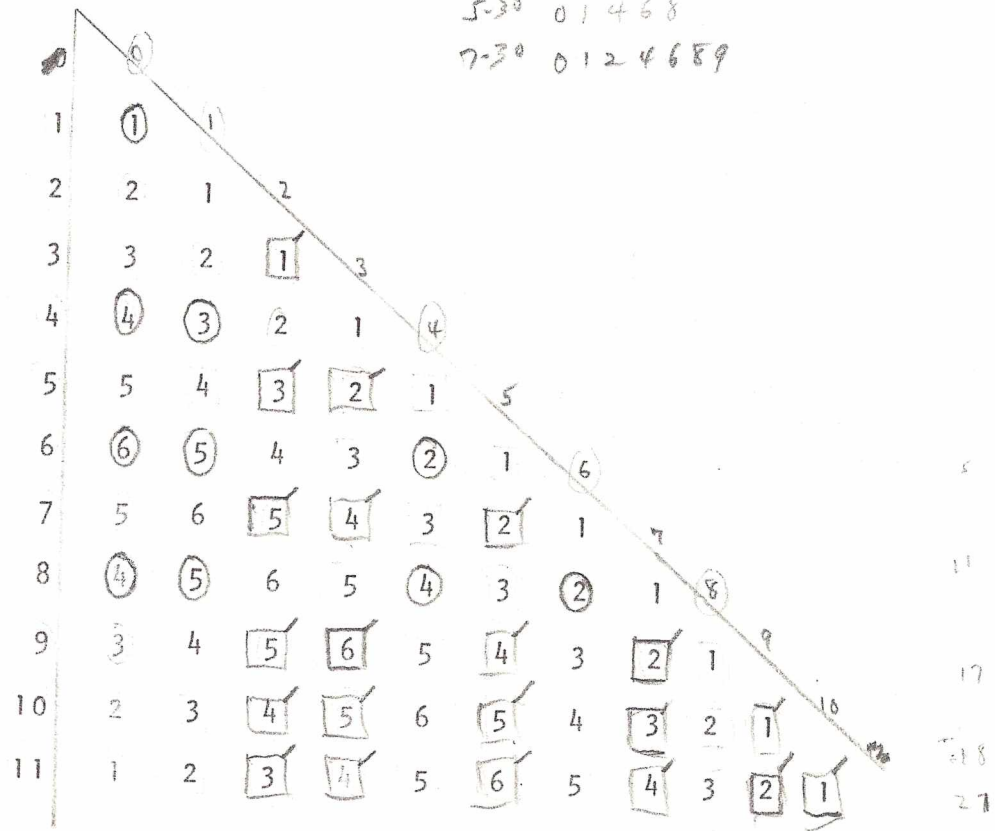


5-30 0 1 4 6 8  
 7-30 0 1 2 4 6 8 9

12  
 20  
 $\frac{20}{2}$



Pairs non-equivalent

For every pair  $\{a, b\}$  in  $S$  there is no pair  $\{c, d\}$  in  $\bar{S}$  such that  $c = a$  and  $d = b$ .  
 I.e., the  $S$  and  $\bar{S}$  are disjoint, by definition of complement

Note that boxes are ~~available~~ <sup>only available</sup> numbers not in row or column of circles

12 IC <sub>1</sub>	- 1 =	11	3
12 IC <sub>2</sub>	- 2 =	10	4
12 IC <sub>3</sub>	- 1 =	11	3
12 IC <sub>4</sub>	- 3 =	9	5
12 IC <sub>5</sub>	- 2 =	10	4
6 IC <sub>6</sub>	- 1 =	5	2

# PROBLEM-SET

Relation betw. pitch-sets and interval-sets

Investigate (theorem?)

Properties of set of integers which form equivalent difference with a third integer.

Examples:	6		
	5	7	$5+7=0 \pmod{12}$
	7		
	5	9	$5+9=2 \pmod{12}$
	8		
	5	11	$5+11=4 \pmod{12}$
	9		
	7	11	$7+11=8 \pmod{12}$
	10		
	9	11	$9+11=8 \pmod{12}$

Sum of the 3 integers congruent to . . .

	6	5	4	3	2	1	0	11	10	9	8	7	6	5	4	3	2	1	0				
5	7	4	6	3	5	2	4	1	3	0	2	11	1	6	8	7	9	8	10	9	11	10	0
4	8	3	7	2	6	1	5	0	4	0	6	10	2	5	9	6	10	9	11	8	0		
3	9	2	8	1	7	0	6					9	3	4	10	5	11	6	0				
2	10	1	9	0	8							8	4	3	11	4	0						
1	11	0	10									7	5	2	0	7	5	6	0				
												6	6										
	0	10	8	6	4	2	0	2	4	6	8	10											
	6	3	0	9	6	3	0	9	0	3	6	9											
	6	3	0	9	6	3	0	9	0	3	6	9											

SUMS OF 3 INTEGERS  
(0? check) etc

# FUNCTIONAL RELATION OF ODD + EVEN IC'S

INTERVAL-SETS: VECTOR STRUCTURE, IC-PC MAPPINGS, ETC.

INVESTIGATE THE RELATION BETWEEN THE SUMS AND PRODUCTS OF A SET OF IC'S

EXAMPLE:

ALSO DIVISION BY #(IC<sub>6</sub>)

*Increase-related IC's*

ADDITION TABLE (If IC<sub>1</sub> and IC<sub>2</sub> -- DISJOINT -- Then IC<sub>3</sub>)

+	1	2	3	4	5	6
1	2	3	4	5	6	5
2	3	4	5	6	5	4
3	4	5	6	5	4	3
4	5	6	5	4	3	2
5	6	5	4	3	2	1
6	5	4	3	2	1	0

*Should include zero*

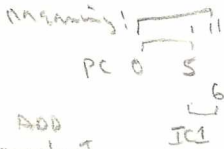
MULTIPLICATION TABLE

*	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	4	2	0
3	3	6	3	0	3	6
4	4	4	0	4	4	0
5	5	2	3	4	1	6
6	6	0	6	6	6	0

*Should include zero*

ODD AND EVEN (ALL)  
EVEN  
ODD AND EVEN (NO 1, 2, 4, 5)  
EVEN (NO 2, NO 6)  
ODD AND EVEN (ALL)  
EVEN (NO 2, NO 4)

$1 = 5 + 6$



ADD  
incident  
to form 6,  
Δ also found

*Meaning of multiplication table*

PC: 0 5 10 3 8 1

$(1 = 1 \cdot 1 = 5 \cdot 5)$



*5 successive 5's  
yield 1 IC<sub>1</sub>*

SYSTEMATICS

PROPERTIES OF VECTORS: DISTRIBUTION OF ODD AND EVEN IC'S

#(S)	#(IC)	' IC				PC	
		O		E		O	E
3	3	E	O	O	E	O	(OR) 0
4	6 (OR) 0	E	O	E	O	E	E
5	10	E	O	E	O	E	O
6	15 (OR) 0	E	O	O	E	E	O

I. E.,  $\forall \#(S) = 3$ , the number of odd IC's with odd Cardinality is even, etc.

CONSIDER PARTITIONS OF #(S) AND #(IC)

PROVE:

THE CARDINAL NUMBER OF ODD INTERVALS IN A HEXACHORD  
IS THE INVERSE OF THE NUMBER OF EVEN INTEGERS IN THE PC SET.  
(mod 12)

AFTER BABBITT JMT P.80

A HEXACHORD AND ITS COMPLEMENT CONTAIN 30 INTERVALS.  
THE REMAINING 36 INTERVALS OCCUR BETWEEN THE HEXACHORDS  
AND ~~THE~~ ARISE FROM THE  $6 \times 6 = 36$  RELATIONS

Interval Set - Subset - Superset Relation  
 $S = [0, 1, 2, \dots, 11]$

SAVE FOR  
 MAIN  
 NOTEBOOK

Mapping of SXS <sup>into</sup> S : an arbitrary  
operation : difference

-	0	1	2	3	4	5	6	7	8	9	10	11
0	0	1	2	3	4	5	6	7	8	9	10	11
1	1	0	1	2	3	4	5	6	7	8	9	10
2	2	1	0	1	2	3	4	5	6	7	8	9
3	3	2	1	0	1	2	3	4	5	6	7	8
4	4	3	2	1	0	1	2	3	4	5	6	9
5	5	4	3	2	1	0	1	2	3	4	5	6
6	6	5	4	3	2	1	0	1	2	3	4	5
7	7	6	5	4	3	2	1	0	1	2	3	4
8	8	7	6	5	4	3	2	1	0	1	2	3
9	9	8	7	6	5	4	3	2	1	0	1	2
10	10	9	8	7	6	5	4	3	2	1	0	1
11	11	10	9	8	7	6	5	4	3	2	1	0

Since SXS is the set of ordered 2-sets  
 subsets, there are 24 IC<sub>1</sub>, 24 IC<sub>2</sub>, etc.

but 12 IC<sub>6</sub>

operation difference is commutative - with  
 symmetry about main axis

SAVE

Equal distribution — as counterpart of unique entries

4-215/4-229	111111	8-215/8-229	555552
<del>5-11</del>	<del>222220</del>	<del>7-11</del>	<del>444441</del>
<del>6-11</del>			

4 or more entries same  
(must hold for couple)

	4-4	211110	8-4	655552
	4-11	121110	8-11	565552
	4-14	111120	8-14	555562
max 3	4-28	004002	8-28	448444
	5-11	222220	7-11	444441
	5-212/236	222121	7-212/236	444242
max 6	5-15	220222	7-15	442443
	5-218/28	212221	7-218/28	434442
max 3,6	5-31	114112	7-31	336333
	<del>5-236</del>			

Examples

6-5 422232

Schoenberg Examples

6-210/39 333321

6-211/40 333231

Schoenberg Examples

6-213/42 324222

6-18 322242

6-23/45 234222

6-24/46 233331

max 3 6-27 225222

Stravinsky, Pistonella

6-228/249 224322

6-229/250 224232

Stravinsky, Pistonella

6-30 224223

Chord

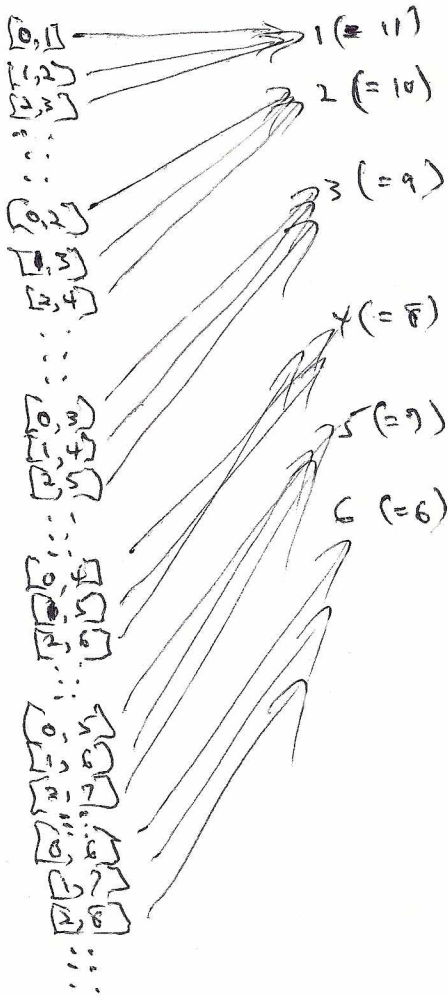


# Interval Set (Set-Set) Relation

Section 2.0.

SAVE  
FOR  
MAIN  
NOTEBOOK

Mapping of  $S \times S$  into  $S$   
is a binary operation in  $S$   
defined equiv. relation



? Omissions - to be checked against set list  
 - should be at least one of each cardinal number?

List of sets with Max IC (s)

Max 1

- [2-1]
- 3-1 / 9-1
- 4-1 / 8-1
- 5-1 / 9-1
- 6-1

Max 2, 4, 6

- ~~2-2~~
- 5-33
- 6-35 - only hex with no ICs

Max 2

- [2-2]
- 3-6
- 4-21 / 8-21

Max 3, 6

- 3-10
- 5-31 - Permit

54

Max 3

- [2-3]
- 4-28 / 8-28
- 6-27

Max 4

- [2-4]
- 3-12 / 9-12
- 4-19 / 8-19
- 4-24 / 8-24 (Wagon Wheel)
- 5-21 / 8-21 Burg. Ch. Pieces (also 9-21) - Busoni?
- 6-20 - op. 41 Naxos - only hex. with no ICs

Max 5

- [2-5]
- 3-9 / 9-9
- 4-23 / 8-23
- 5-35 / 9-35
- 6-32

Max 6

- [2-6]
- 3-5 / 9-5
- 3-8 / 9-8
- 4-9 / 8-9
- 4-25 / 8-25
- ✓ 5-7 / 7-9, Ives Chromatic Lattice List, Unstim
- 5-15 / 9-15
- 5-19 - Wagon Wheel 7-19, Cowell's Tiger
- 5-28 - 7-28 is one of Busoni's books
- 6-7
- 6-30



EXAMINE ■ PROPERTIES OF INTERVAL-VECTORS IN  
TERMS OF CARDINALITY OF SET AND MAX-MIN INTERVAL-  
CORRESPONDENCES (GROUP THEORY?)

---

Rank-ordering (relative frequency) of each IC in pairs-set  
vectors

---

Why is IC4 present in every 5-vector and 6-vector?  
— partitions of 12

---

Algorithm for discovering common adjacent segments  
between 2 sets on basis of interval-contrast (Beach thesis)

---

Quaternary vectors for:

OBVERSE — Exchange of entries

COMPLEMENT OBVERSE

Exchange of retrograde

(See logic text  
for usage)

## PROPERTIES OF VECTORS

Four sets have interval vectors such that ~~the~~  
no entry/element is the same as any other  
("IC uniqueness with respect to cardinal number")

- 6-1 (max 1)
- 6-32 (max 5) vector of all-interval hexachord set used by Berg in Storm song
- 7-1 (max 1)
- 7-35 (max 5) major scale (and modes)

*all have max property*

Relation between cardinal number and number of intervals of a set.

Example:  $a, b$   
 $1, 2$  1 difference  
 $a, b, c$   
 $1, 2, 3,$  3 differences  $(=2+1)$   
 $a, b, c, d,$   
 $1, 2, 3, 4,$  6 differences  $(=3+2+1)$

Thus, if  $\#(S) = n$

~~xxxx~~ the number of intervals  $m$  is the sum of the arithmetic ~~sequence~~ sequence  $1, 2, \dots, n-1$

or:  $\frac{(n-1)^2 + n-1}{2}$

[also,  $\frac{n^2 - n}{2}$ ]

Table:

$\#(S)$	M
1	0] - trivial
2	1
3	3
4	6
5	10
6	15
7	21
8	28
9	36
10	45
11	55
12	66

(additional)

Three trivial cases:

$\#(S) = 12$  [12 12 12 12 12 6] i.e., same vector for any 12-element set  
 $\#(S) = 11$  [10 10 10 10 10 5]  
 $\#(S) = 0$  [0 0 0 0 0 0]

To compute vector of complement:

Example: a set with cardinality 2 and a set with cardinality 10

$10 - 2 = 8$  difference of cardinal numbers  
 $45 - 1 = 44$  difference of number of intervals  
 $5 \times 8 = 40$  add 8 to first five IC's  
 $40 + 8/2 = 44$  add 8/2 to last IC (IC6)  
 $44 + 1 = 45$  1 IC already contains a 1