12
20
24
[illegible]

5-30 0 1 4 6 8
7-30 0 1 2 4 6 8 9

0	0												
1	1	1											
2	2	1	2										
3	3	2	$\boxed{1}$	3									
4	4	3	2	1	4								
5	5	4	$\boxed{3}$	$\boxed{2}$	1	5							
6	6	5	4	3	2	1	6						5
7	5	6	$\boxed{5}$	$\boxed{4}$	3	$\boxed{2}$	1	7					
8	4	5	6	5	4	3	2	1	8				11
9	3	4	$\boxed{5}$	$\boxed{6}$	5	$\boxed{4}$	3	$\boxed{2}$	1	9			17
10	2	3	$\boxed{4}$	$\boxed{5}$	6	$\boxed{5}$	4	$\boxed{3}$	2	$\boxed{1}$	10		18
11	1	2	$\boxed{3}$	$\boxed{4}$	5	$\boxed{6}$	5	$\boxed{4}$	3	$\boxed{2}$	$\boxed{1}$		27

Pairs non-equivalent

For every pair {a, b} in S there is no pair {c, d} in $\bar{S}$ such that c = a and d = b. I.e., ~~then~~ S and $\bar{S}$ are disjoint, by definition of complement

Note that boxes are ~~only available~~ only available numbers not in rows or column of circles

12 IC_1	− 1	=	11		3
12 IC_2	− 2	=	10		4
12 IC_3	− 1	=	11		3
12 IC_4	− 3	=	9		5
12 IC_5	− 2	=	10		4
6 IC_6	− 1	=	5		2

PROBLEM-SET

Relation betw. pitch-sets and interval-sets

Investigate (theorem?)

Properties of set of integers which form equivalent difference with a third integer.

Examples:

6		
5	7	5+7=0(mod 12)
7		
5	9	5+9=2(mod 12)
8		
5	11	5+11=4(mod 12)
9		
7	11	7+11=~~xxx~~ 6(mod 12)
10		
9	11	9+11=8(mod 12)

Sum of the 3 integers congruent to . . .

	6	5	4	3	2	1	0 (4·6)	7	8	9	10	11
	5 7	4 6	3 5	2 4	1 3	0 2	11 1	6 8	7 9	8 10	9 11	10 0
	4 8	3 7	2 6	1 5	0 4		10 2	5 9	6 10	7 11	8 0	
	3 9	2 8	1 7	0 6			9 3	4 10	5 11	6 0		
	2 10	1 9	0 8				8 4	3 11	4 0			
	1 11	0 10					7 5	2 0				
							6 6					
	0	10	8	6	4	2	0	2	4	6	8	10
SUMS OF 3 Integers	6	3	0	9	6	3	0	9	0	3	6	9
(°7 chord/one)	6	3	0	9	6	3	0	9	0	3	6	9

FUNCTIONAL RELATION OF ODD + EVEN IC's

INTERVAL-SETS: VECTOR STRUCTURE, IC-PC MAPPINGS, ETC.

INVESTIGATE THE RELATION BETWEEN THE SUMS Σ AND PRODUCTS Π OF A SET OF IC'S

EXAMPLE:

ALSO DIVISION BY #(IC_6)

Inverse-related IC's

ADDITION TABLE (If IC_1 and IC_2 -- DISJOINT -- Then IC_3)

Should include zero

+	1	2	3	4	5	6
1	2	3	4	5	6	5
2	3	4	5	6	5	4
3	4	5	6	5	4	3
4	5	6	5	4	3	2
5	6	5	4	3	2	1
6	5	4	3	2	1	0

MULTIPLICATION TABLE

Should include zero

·	1	2	3	4	5	6	
1	1	2	3	4	5	6	ODD AND EVEN (ALL)
2	2	4	6	4	2	0	EVEN
3	3	6	3	0	3	6	ODD AND EVEN (NO 1,2,4,5)
4	4	~~6~~4	0	4	4	0	EVEN (NO 2, NO 6)
5	5	2	3	4	1	6	ODD AND EVEN (ALL)
6	6	0	6	4	6	0	EVEN (NO 2, NO 4)

1 = 5 + 6

meaning!

PC 0 5 6

IC1

ADD one element to form 6, 1 also formed

meaning of multiplication here

PC: 0 5 10 3 8 1

(1 = 1·1 = 5·5)

1.5 2.5 3.5 4.5 5.5

5 successive 5ths yield 1 IC_1

SYSTEMATICS

PROPERTIES OF VECTORS: DISTRIBUTION OF ODD AND EVEN IC'S

#(S)	#(IC)	IC: O (O)	IC: O (E)	IC: E (O)	IC: E (E)	PC: O	PC: E
3	3	E	O	O	E	E	O
						(OR) O	E
4	6	E	O	E	O	E	E
	(OR)	O	E	O	E		
5	10	E	O	E	O	E	O
6	15	E	O	O	E	E	E
	(OR)	O	E	E	O	(OR) O	O

I.E., if #(S) = 3, the ~~cardinal~~ number of odd IC's with odd cardinality is even, etc.

CONSIDER PARTITIONS OF #(S) AND #(IC)

PROVE:

THE CARDINAL NUMBER OF ODD INTERVALS IN A HEXACHORD

(mod 12)
IS THE INVERSE OF THE NUMBER OF EVEN INTEGERS IN THE PC SET.

AFTER BABBITT JMT P.80

A HEXACHORD AND ITS COMPLEMENT CONTAIN~~S~~ 30 INTERVALS.
THE REMAINING 36 INTERVALS OCCUR BETWEEN THE HEXACHORDS
AND ARISE FROM THE 6 X 6=36 RELATIONS

Interval-Set - Pitch-Set Relation
$S = [0, 1, 2, \ldots 11]$

SAVE FOR MAIN NOTEBOOK

Mapping of $S \times S$ ~~into~~ onto S: a binary operation: difference

−	0	1	2	3	4	5	6	7	8	9	10	11
0	0	1	2	3	4	5	6	7	8	9	10	11
1	1	0	1	2	3	4	5	6	7	8	9	10
2	2	1	0	1	2	3	4	5	6	7	8	9
3	3	2	1	0	1	2	3	4	5	6	7	8
4	4	3	2	1	0	1	2	3	4	5	6	7
5	5	4	3	2	1	0	1	2	3	4	5	6
6	6	5	4	3	2	1	0	1	2	3	4	5
7	7	6	5	4	3	2	1	0	1	2	3	4
8	8	7	6	5	4	3	2	1	0	1	2	3
9	9	8	7	6	5	4	3	2	1	0	1	2
10	10	9	8	7	6	5	4	3	2	1	0	1
11	11	10	9	8	7	6	5	4	3	2	1	0

Since $S \times S$ is the set of ordered ~~pairs~~ 2-note subsets, there are 24 IC_1, 24 IC_2, etc.
\ but 12 IC_6

Operation difference is commutative — note symmetry about main axis

SAVE

Equal distribution — as counterpart of unique entries

	4-Z15/4-Z29	111111	8-Z15/8-Z29	555553
	~~5-11~~	~~222220~~	~~7-11~~	~~444441~~
	~~C7-11~~			

4 or more entries same
(must hold for comple.)

	4-4	211110	8-4	655552
	4-11	121110	8-11	565552
	4-14	111120	8-14	555562
max 3	4-28	004002	8-28	448444
	5-11	222220	7-11	444441
	5-Z12/Z36	222121	7-Z12/Z36	444342
max 6	5-15	220222	7-15	442443
	5-Z18/Z38	212221	7-Z18/Z38	434442
max 3,6	5-31	114112	7-31	336333
	~~5-Z36~~			
Example	6-5	422232		
Schoenberg Example	6-Z10/Z39	333321		
	6-Z11/Z40	333231		
Schoenberg Example	6-Z13/Z42	324222		
	6-18	322242		
	6-Z23/Z45	234222		
	6-Z24/Z46	233331		
max 3	6-27	225222		
Stravinsky Petroushka	6-Z28/Z49	224322		
	6-Z29/Z50	224232		
Stravinsky, Petroushka Chord	6-30	224223		

Interval-Set Petal-Set Relation

SAVE FOR MAIN NOTEBOOK

Section 2.0.

Mapping of $S \times S$ into S

is a binary operation in S

Defined equiv. relation

[0,1], [1,2], [2,3], ...	1 (= 11)
[0,2], [1,3], [2,4], ...	2 (= 10)
[0,3], [1,4], [2,5], ...	3 (= 9)
[0,4], [1,5], [2,6], ...	4 (= 8)
[0,5], [1,6], [2,7], ...	5 (= 7)
[0,6], [1,7], [2,8], ...	6 (= 6)

? Omissions – to be checked against set list
– should be at least one of each cardinal number?

List of sets with Max IC (s)

Max 1
[2-1]
3-1 /9-1
4-1 /8-1
5-1 /7-1
6-1

Max 2
[2-2]
3-6
4-21/8-21

Max 3
[2-3]
4-28/8-28
6-27

Max 4
[2-4]
3-12 /9-12
4-19 /8-19
4-24 /8-24 (Wozzeck chord)
5-21/~~7-21~~ Berg Cl. Pieces (also 7-21) – Busoni?
6-20 – opposite Magnum – only hex. with no IC2

Max 5
[2-5]
3-9 /9-9
4-23 /8-23
5-35 /7-35
6-32

Max 6
[2-6]
3-5/9-5
3-8/9-8
4-9 /8-9
4-25/8-25
5-7 / 7-7, Ives Chromatimelodtune Liszt, Unstern
5-15/7-15
5-19 – Wozzeck Lullaby 7-19, Cowell's Tiger
5-28 – 7-28 is one of Busoni's scales
6-7
6-30

Max 2,4,6
~~6-22~~
5-33
6-35 – only hex with no IC5

Max 3,6
3-10
5-31 – Pierrot

54

NOTES ON

GENERAL FUNCTIONAL RELATION OF IC'S FOR 3-NOTE PC SETS:

REVERSIBLE?

A PARTITION:

2 IC'S DETERMINE THE 3RD IC FOR A 3-NOTE SET
(AND IN GENERAL, THE SELECTION OF N IC'S DETERMINES VECTOR)

FIRST SUBSCRIPT INDICATES CLASS, SECOND INDICATES CARDINALITY OF THAT CLASS.

1. SELECT ANY TWO ODD IC'S   POSSIBILITIES ARE:
1 AND 3
1 AND 5
3 AND 5

(ODD IC'S DETERMINE EVEN)

2. GIVEN IC_{i_p}, IC_{j_q} . Determine IC_{k_r}

Solution: (by operating on subscripts)

$k = ip + jq$

$r = p + q/2$

generalize "constant" 2, actually a variable representing the number of second subscripts.

nb. $k = ip + jq'$ determines a different IC

THIS FUNCTION IS _ CONTROLLED BY the subgroup 0 4 8 and its left cosets

Thus, the relation between IC's can be read from the left cosets

(to be worked out)

"PARITY" (Kemeny p. 71)

odd or even characteristic of an integer

EXAMINE [redacted] PROPERTIES OF INTERVAL-VECTORS IN TERMS OF CARDINALITY OF SET AND MAX-MIN INTERVAL-CORRESPONDENCES (GROUP THEORY?)

Rank-ordering (relative frequency) of each IC in prime-set vectors

---

Why is IC4 present in every 5-vector and 6-vector?

— partitions of 12

~~Algorithm for discovering common adjacent segments between 2 sets on basis of interval-content (Beach thesis)~~

Examine vectors for;

OBVERSE — exchange of entries

COMPLEMENT OBVERSE

exchange + retrograde

(See logic text for usage)

PROPERTIES OF VECTORS

Four sets have interval~~x~~ vectors such that ~~xhx~~
no entry/element is the same as any other
("IC uniqueness with respect to cardinal number")

6-1 (max 1)
6-32 (max 5) vector of all-interval ~~set~~ hexachord used by Berg in Storm song
7-1 (max 1)
7-35 (max 5) major scale (and modes)

all have max property

Relation between cardinal number and number of intervals of a set.

Example:
a, b
1,2 1 difference
a, b, c
1,2,3, 3 differences (=2+1)
a, b, c, d,
1,2,3,4, 6 differences (=3+2+1)

Thus, if #(S) = n

~~xfor~~ the number of intervals m is the sum of the arithmetic [redacted] sequence 1,2,. . . , n-1

or: $\frac{(n-1)^2 + n-1}{2}$

[also, $\frac{n^2 - n}{2}$]

Table:

#(S)	M
[1	0] - trivial
2	1
3	3
4	6
5	10
6	15
7	21
8	28
9	36
10	45
11	55
12	66

Three (additional) trivial cases:

#(S) = 12 [12 12 12 12 12 6] i.e., same vector for any 12-element set

#(S) = 11 [10 10 10 10 10 5]

#(S) = 0 [0 0 0 0 0 0]

To compute vector of complement:

Example: a set with cardinality 2 and a set with cardinality 10

10 - 2 = 8	difference of cardinal numbers
45 - 1 = 44	difference of number of intervals
5 x 8 = 40	add 8 to first five IC's
40 + 8/2 = 44	add 8/2 to last IC (IC6)
44 + 1 = 45	1 IC already contains a 1